Programme and Abstracts

The 5th International Combinatorics Conference

Monash University, Melbourne, Australia December 4–9, 2017



Combinatorial Mathematics Society of Australasia (CMSA)



School of Mathematical Sciences, Monash University

All plenary talks will be in Lecture Theatre E3. Contributed talks will be in Lecture Theatres E3, E5, E6 depending on whether they are in the left, middle or right column of the timetable respectively.

Sunday 3 December

4:00-7:00

Registration and Welcome Party

Ground floor of the maths building, 9 Rainforest Walk

Monday

8:30	Registration (Outside Lecture Theatre E3)		
9:00	Daniela Kühn Hypergraph F-designs exist for arbitrary F (p11) Chair: Wanless		
10:00	Mou Gao Hamiltonian prisms under toughness conditions (p26) Chair: Scott	Edward Lee Exact Algorithms via Multivariate Subroutines (p35) Chair: Isaev	Augustine O. Munagi Inplace multiplicity identities for integer compositions (p42) Chair: Best
10:25	Morning Tea		
10:50	Brendan McKay Counting regular graphs and not-so-regular graphs (p12) Chair: Isaev		
11:50	Shohei Satake Distance sets over finite spaces and finite Euclidean graphs (p50) Chair: Royle	Florian Lehner Bounding the cop-number of a graph in terms of its genus (p35) Chair: Simpson	Brian Alspach On (1,2)-Invariant Graphs (p16) Chair: Herke
12:15	Martin Bachraty Approaching the Moore bound for diameter 3 by Cayley graphs (p18) Chair: Royle	Douglas B. West The Slow-Coloring Game (Online Sum-Paintability) (p61) Chair: Simpson	Hao Chuien Hang Hamilton Path Decompositions of Complete Multipartite Graphs (p29) Chair: Herke
12:40		Lunch	
2:00	Charles J. Colbourn Asymptotic and Constructive Bounds for Hash Families (p8) Chair: Horsley		
3:00	Amy Glen On a generalisation of the Fibonacci word to an infinite alphabet (p27) Chair: Wanless	Mujiangshan Wang The 1-good-neighbor connectivity and diagnosability of Cayley graphs generated by complete graphs (p60) Chair: Glasby	Nhan Bao Ho Playing impartial games: from how to win to how to lose (p31) Chair: Farr
3:25	Afternoon Tea		
3:50	Yanli Qin Cubic edge-transitive bi-p-metacirculants (p47) Chair: Giudici	Daniel R. Hawtin Designs and Minimal 2-Neighbour-Transitive Codes (p30) Chair: Glasby	Johann A. Makowsky On weakly distinguishing graph polynomials (p37) Chair: Noble
4:15	Luke Morgan Cayley digraphs and Cayley index (p41) Chair: Giudici	Tatsuya Maruta Nonexistence of some linear codes over the field of order four (p39) Chair: Glasby	Ruixue Zhang Properties of chromatics polynomials of hypergraphs not held for chromatic polynomials of graphs (p64) Chair: Noble
4:40	Yian Xu On Constructing Normal and Non-Normal Cayley Graphs (p63) Chair: Giudici	Xiao-Nan Lu Locating arrays with error correcting ability (p37) Chair: Glasby	Kai Siong Yow Characterisations of Extended Tutte Invariants for Alternating Dimaps (p64) Chair: Noble

CMSA AGM (in E3)

5:05

Tuesday (Including celebration of Tutte centenary)

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9:00	Le Anh Vinh Expanders (p14)	in finite fields	Chair: Payne
10:00	Deryk Osthus A characterization of testable hypergraph properties (p45) Chair: Gao	Jonathan Klawitter Spaces of phylogenetic networks (p34) Chair: Mayhew	Kirsten Nelson An equivalence relation on shift sequences (p43) Chair: Conder
10:25		Morning Tea	
10:50	Alex Scott Graphs of larg (p12)	e chromatic number	Chair: Wood
11:50	Mikhail Isaev Subgraph counts for dense graphs with specified degrees (p32) Chair: Gao	Kevin Hendrey When graphs are too small to be avoided (p30) Chair: Simpson	Guillermo Pineda-Villavicencio Connectivity of cubical polytopes (p45) Chair: Verret
12:15	Haya Aldosari Enumeration of sparse uniform hypergraphs with forbidden edges (p16) Chair: Gao	Sheng Bau Reductions of 3-connected graphs with minimum degree at least four (p19) Chair: Simpson	David Yost Polytopes with few edges are rare (p63) Chair: Verret
12:40		Lunch	
2:00	Marston Conder Cayley (p9)	graphs revisited	Chair: Dietrich
3:00	Shuya Chiba On degree sum conditions for 2-factors with a prescribed number of cycles (p21) Chair: Isaev	Georgina Liversidge Strongly Sequenceable Groups (p36) Chair: Colbourn	Rinovia Simanjuntak The Multiset Dimension of Graphs (p53) Chair: Hendrey
3:25		Afternoon Tea	
3:50	Marc Demange Online Colouring Problems in Overlap Graphs and their Complements (p22) Chair: Greenhill	Masatake Hirao Comparison of frame potentials of random point configurations on the sphere (p31) Chair: Bamberg	Kaiyue Duan Efficient Security Repairing in Multi-Cloud Storage using Latin Square Autotopism Secret Sharing (p23) Chair: Herke
4:15	Tim E. Wilson Path-Dependent Colour Schemes (p61) Chair: Greenhill	Günter Steinke Almost 2-transitive finite elation Laguerre planes (p53) Chair: Bamberg	Joanne L. Hall Isomorphism classes of difference covering arrays and pseudo-orthogonal Latin squares. (p29) Chair: Herke
4:40	David R. Wood Bad News for Chordal Partitions (p62) Chair: Greenhill	Duy Ho Finite Minkowski planes of type 17 with respect to homotheties (p31) Chair: Bamberg	Rebecca J. Stones Balanced equi-n-squares: a generalization of Latin squares (p54) Chair: Herke
5:05	Akira Kamibeppu On a sufficient condition of a graph with boxicity at most its chromatic number (p33) Chair: Greenhill	Muhammad Adib Surani The Bisection Width of the Levi Graph of $PG(2,q)$ (p55) Chair: Bamberg	Trent G. Marbach Covers and partial transversals of Latin squares. (p39) Chair: Herke
5:30	A.N.M. Salman The Rainbow Connection Number of Join of Some Graphs (p48) Chair: Greenhill	Mark Ioppolo Symmetries of Codes in Affine Geometries (p32) Chair: Bamberg	Jayama Mendis Cycle structures of certain autoparatopisms of Latin Squares (p40) Chair: Herke

Wednesday

9:00	Paul Seymour Gyárfás-Se (p13)	umner meets Erdős-Hajnal	Chair: Morgan
10:00	Michael Albert Wilf-collapse in permutation classes (p15) Chair: Royle	Binzhou Xia Cyclotomic difference sets (p62) Chair: Praeger	Marcel Adonis Haddad k-center in graphs with uncertainty on vertices (p28) Chair: Cavenagh
10:25	Jinge Li Wilf equivalences in some small permutation classes (p36) Chair: Royle	Don Taylor Hadamard difference sets of order 256 (p57) Chair: Praeger	Harald Bögeholz Calculating Connected Components in Huge Graphs by Randomised Contraction (p20) Chair: Cavenagh
10:50	Morning Tea		
11:30	Buses leave for Excursion		
6:00	(Approx) Return from Excursion		

Thursday

9:00	Joanna Fawcett Homoger (p10)	neity in graphs and partial linea	ar spaces Chair: Dietrich
10:00	Robert Bailey On the metric dimension of incidence graphs (p18) Chair: Colbourn	Adam Mammoliti The Erdős-Ko-Rado Theorem, Generalisations and Beyond (p38) Chair: Taylor	Boon Leong Ng Relationships between independence and matching coefficients of trees (p44) Chair: Glen
10:25		Morning Tea	
10:50	Maria Chudnovsky 4-coloring graphs with no induced 6-vertex path (p8) Chair: Liebenau		-
11:50	Denny Riama Silaban Restricted Size Ramsey Number for $2K_2$ versus Dense Connected Graphs (p52) Chair: Mendis	Stephen Glasby Norman involutions and tensor products of unipotent Jordan blocks (p27) Chair: Mayhew	Milad Ahanjideh Nowhere-zero 3-flows in Cayley graphs on certain finite groups (p15) Chair: Liebenau
12:15	Srashti Dwivedi On the two-colour Rado number for $\sum_{i=1}^{m} a_i x_i = c$ (p23) Chair: Mendis	Gordon Royle On the structure of thin cubic Lehman matrices (p48) Chair: Mayhew	Tamás Mészáros Exploring the projective norm graph (p40) Chair: Liebenau
12:40		Lunch	
2:00	David Eppstein Forbidden Configurations in Discrete Geometry (p9) Chair: Wood		
3:00	Lenny Tevlin Two New Statistics on Square Walks. (p57) Chair: Wood	Jane Tan A generic approach to switching reconstruction (p56) Chair: Taylor	Gabriel Verret Distinguishing number of vertex-transitive graphs of valency 4 (p59) Chair: Guidici
3:25		Afternoon Tea	
3:50	Shoichi Tsuchiya On distance matching extendability of fullerene graphs (p58) Chair: Ellingham	Xiaohong Zhang Point-transitivity of 2-(196, 6, 1) designs (p65) Chair: Horsley	Michał Lasoń White's conjecture and related matroid problems (p34) Chair: Farr
4:15	Atsuhiro Nakamoto $Y\Delta$ -equivalence classes of projective planar maps and rhombus tilings of polygons (p42) Chair: Ellingham	Maryam Tale Masouleh The relations between even block designs as finite groups (p39) Chair: Horsley	Nick Brettell Detachable pairs in 3-connected matroids (p20) Chair: Farr
4:40	Seiya Negami Stable embeddings of graphs on closed surfaces with respect to minimum length and their duals (p43) Chair: Ellingham	Yudhistira Andersen Bunjamin Lessons from the Non-Existence of a 2-Resolvable BIBD(10,15,6,4,2) (p21) Chair: Horsley	Keisuke Shiromoto Matroids and Codes with the Rank Metric (p52) Chair: Farr
5:05	Kengo Enami Re-embedding structures of 3-connected 3-regular planar graphs into non-spherical surfaces (p25) Chair: Ellingham	Chariya Uiyyasathian Group Divisible Designs with $\lambda_1 = 3$ and Larger Second Index (p59) Chair: Horsley	Steve Noble Vf-safe delta-matroids (p44) Chair: Farr
5:30	Yumiko Ohno Triad colorings of triangulations on the torus (p44) Chair: Ellingham	Masanori Sawa Gaussian designs, quasi-Hermite polynomials, Hausdorff-type Diophantine equations (p51) Chair: Horsley	Eric Ould Dadah Andriantiana Subtrees and independent subsets in unicyclic graphs and unicyclic graphs with fixed segment sequence (p16) Chair: Farr

Friday (Anne Street Memorial Day)

9:00	Barbara Maenhaut 50 ye (p11)	ears of the Oberwolfach Problem	n Chair: Horsley
10:00	Cheryl E Praeger Composition lengths of finite groups (p46) Chair: Horsley	David Ellison Firefighter Problem on Trees (p25) Chair: Taylor	Suhadi Wido Saputro On local metric dimension of Mycielski graph (p50) Chair: Gill
10:25		Morning Tea	
10:50	Catherine Greenhill Star decompositions of random regular graphs (p28) Chair: McLeod	Michael Giudici 2-arc-transitive digraphs (p26) Chair: Zhou	I Wayan Palton Anuwiksa Distance Magic Labelings of Distance-Regular Graphs (p17) Chair: Hall
11:15	Nicholas Cavenagh Balanced diagonals in frequency squares (p21) Chair: McLeod	Heiko Dietrich A quick survey on coclass graphs (p22) Chair: Zhou	Yeva Fadhilah Ashari (K_2, H) -Magic labelings of Graphs $(p17)$ Chair: Hall
11:40	Sara Herke The worst case for k-independence (p30) Chair: McLeod	Vedrana Mikulić Crnković On combinatorial structures constructed from unitary groups U(3,q), q=3,4,5,7	Aholiab Tegar Tritama Distance Antimagic Labeling of Graph Products (p58) Chair: Hall
12:05	Daniel Horsley Sperner partition systems (p32) Chair: McLeod	(p41) Chair: Zhou Nemanja Poznanović A Study of Imprimitive Half-Arc-Transitive Graphs (p46) Chair: Zhou	Kiki Ariyanti Sugeng On Inclusive and non Inclusive Distance Vertex Irregular Labelings (p54) Chair: Hall
12:30		Lunch	
2:00	Jacob Fox Arithmetic regulation (p10)	ularity, removal, and progressic	ons Chair: Gao
3:00	Paul Leopardi Yet another database of strongly regular graphs (p36) Chair: Bailey	Robert Šámal Exponentially many nowhere-zero Z_3 -, Z_4 -, and Z_6 -flows (p49) Chair: Best	Srinibas Swain The Most Frequent Connected Induced Subgraph problem (p55) Chair: Glen
3:25		Afternoon Tea	
3:50	Sanming Zhou Perfect Codes in Cyclotomic Graphs (p65) Chair: Bailey	Joanna Ellis-Monaghan Graph embedding and the complexity of the DNA reporter strand problem (p24) Chair: Farr	Thomas Selig Permutations and recurrent configurations of the Abelian sandpile model on Ferrers graphs (p51) Chair: Albert
4:15	Bin Jia Imprimitive symmetric graphs which are not multicovers of their quotients (p33) Chair: Bailey	Terukazu Sano A characterization of tree-tree quadrangulations on closed surfaces (p49) Chair: Farr	Murray Tannock Patterns in Arc System (p56) Chair: Albert
4:40	John Bamberg Cages arising from generalised polygons.	Raiji Mukae Purity of projective planar graphs	M Rana Combinatorics of tenth-order mock theta functions

Conference Dinner

6:30

Saturday

9:30	Balázs Szegedy On the eige matrices (p13)	envectors of random regular graphs and random Chair: Wormald
10:30	Morning Tea	
11:00	Bruce Reed Some variants of the Erdős-Sós Conjecture (p47) Chair: Greenhill	
11:25	Mark Ellingham Toughness and treewidth (p24) Chair: Greenhill	
11:50	Ian Wanless Congruences for Permanents (p60) Chair: Greenhill	
12:05		Conference ends
8:00		Survivors' party

Abstracts of invited talks

4-coloring graphs with no induced 6-vertex path

Maria Chudnovsky*, Sophie Spirkl, Mingxian Zhong
*Princeton University

The problem of testing if a graph can be colored with a given number k of colors is NP-complete for every k > 2. But what if we have more information about the input graph, namely that some fixed graph H is not present in it as an induced subgraph? It is known that the problem remains NP-complete if H is not a subgraph of a path, and the question has been extensively studied in the case when H is a path. Only two cases remain open: 3-coloring a graph with no induced t-vertex path when $t \ge 8$, and 4-coloring a graph with no induced 6-vertex path. Recently in joint work with Sophie Spirkl and Mingxian Zhong we settled one of these two problems and found a polynomial-time algorithm to determine if a graph with no induced 6-vertex path admits a 4-coloring (and finds such a coloring if it exists). In this talk we will survey what is known on the topic, and discuss the main ideas of our recent algorithm.

(Thursday 10:50)

Asymptotic and Constructive Bounds for Hash Families

Charles J. Colbourn*

*Arizona State University

A perfect hash family PHF(N; k, w, t) is an $N \times k$ array on w symbols, in which in every $N \times t$ subarray, at least one row consists of distinct symbols (and hence separates the t columns). Perfect hash families arise in combinatorial cryptography and in constructions of covering arrays; one wants to minimize the number N of rows given that the PHF has k columns, w symbols, and strength t. Many related types of hash families have been defined, motivated by similar applications. Although direct constructions from codes, designs, finite geometries, and arithmetic sequences are known, the construction of specific hash families needed in applications remains challenging.

Indeed, for specific parameters, it often happens that the best array constructed by direct and recursive methods fails to achieve the size guaranteed by simple probabilistic arguments. Because of this, efficient algorithms to achieve the asymptotic bounds from probabilistic methods are of interest. We explore two asymptotic bounds, one from an elementary probabilistic argument and one via the Lovász Local Lemma. Using the Stein-Lovász-Johnson conditional expectation method for the first, and the Moser-Tardos resampling method for the second, we describe constructive methods for hash families that are easily implemented and prove to be effective for relatively large parameters.

Finally we adapt the random selection to use oversampling, and discuss both the asymptotic and the algorithmic consequences.

(Monday 2:00)

Cayley graphs revisited

Marston Conder*

* University of Auckland

Tutte Centenary Lecturer

A Cayley graph is a graph that encodes some of the structure of a group — giving a representation of part of the multiplication table for the group — but it can also be viewed as a graph on which some group acts regularly (sharply-transitively) on vertices. The notion was introduced by Arthur Cayley in 1878, and developed by Max Dehn for use in studying the word problem for the fundamental groups of hyperbolic surfaces, which in turn led to what is now known as 'geometric group theory'.

Cayley graphs are very easy to understand and construct, but remarkably there is quite a lot about them that remains a mystery, or has become known only recently. For example, when does a Cayley graph have a Hamilton cycle? And how often is the automorphism group of a Cayley graph equal to the group from which it is constructed? And can that happen infinitely often for every valency? When is a Cayley graph not just vertex-transitive, but also arc-transitive? And what about 'bi-Cayley' graphs, where the group has two regular orbits on vertices? Or m-Cayley, when it has m regular orbits on vertices?

These questions can be addressed using a combination of graph theory and group theory (including some of the seminal work of Bill Tutte), and even some topology. In this lecture I'll explain some of the background to them and progress on answering them, and give some recent answers. (Tuesday 2:00)

Forbidden Configurations in Discrete Geometry

David Eppstein*

* University of California, Irvine

We review and classify problems in discrete geometry that depend only on the order-type or configuration of a set of points, and that can be characterized by a family of forbidden configurations. These include the happy ending problem, no-three-in-line problem, and orchard-planting problem from classical discrete geometry, as well as Harborth's conjecture on integer edge lengths and the construction of universal point sets in graph drawing. We investigate which of these properties have characterizations involving a finite number of forbidden subconfigurations, and the implications of these characterizations for the computational complexity of these problems. (Thursday 2:00)

9

Homogeneity in graphs and partial linear spaces

Joanna Fawcett*

*Imperial College London, UK

A graph is homogeneous if any isomorphism between finite induced subgraphs extends to an automorphism of the entire graph. The finite homogeneous graphs have been completely classified, and only a few families of examples arise. There is a natural way of defining homogeneity for a much larger class of incidence structures: partial linear spaces. A partial linear space consists of a set of points and a set of lines such that any two distinct points are contained in at most one line, and every line contains at least two points. In this talk, we will discuss several methods of relaxing the hypothesis of homogeneity for graphs and, more generally, partial linear spaces. (Thursday 9:00)

Arithmetic regularity, removal, and progressions

<u>Jacob Fox</u>*, Lszl Mikls Lovsz, Huy Tuan Pham, Lisa Sauermann, and Yufei Zhao.

*Stanford University

Szemerédi's regularity lemma and its variants are some of the most powerful tools in combinatorics. For example, Szemerédi used an early version in the proof of his celebrated theorem on long arithmetic progressions in dense sets of integers. It has also played a central role in property testing, which concerns finding fast randomized algorithms for distinguishing between objects that have a property from objects that are far from having that property. The regularity lemma roughly says that the vertex set of every graph can be partitioned into a "small" number of parts such that for most of the pairs of parts, the bipartite graph between the pair is quasirandom. A substantial drawback in using the regularity lemma is that it typically gives enormous, tower-type bounds in its various applications. A major program over the last few decades, championed by Szemerédi and others, has been to find alternative proofs of the many applications of the regularity lemma with much better quantitative bounds. We will discuss some major successes of the program, as well as the first examples of applications in which the tower-type bounds which come from applying a regularity lemma is necessary.

(Friday 2:00)

Hypergraph F-designs exist for arbitrary F

Stefan Glock, <u>Daniela Kühn</u>*, Allan Lo and Deryk Osthus

*Birmingham University

We show that given any r-uniform hypergraph F, the trivially necessary divisibility conditions are sufficient to guarantee a decomposition of any sufficiently large complete r-uniform hypergraph into edge-disjoint copies of F. The case when F is complete corresponds to the existence of block designs, a problem going back to the 19th century, which was recently settled by Keevash. In particular, our argument provides a new proof of this result, which employs purely probabilistic and combinatorial methods. We also obtain several further generalizations. (Monday 9:00)

50 years of the Oberwolfach Problem

Barbara Maenhaut*

* The University of Queensland

Anne Penfold Street Memorial Lecturer

The Oberwolfach problem is a graph factorisation problem posed by Gerhard Ringel in 1967 at a conference in Oberwolfach, Germany. For $n \ge 3$ and F a 2-regular graph of order n, the Oberwolfach Problem $\mathrm{OP}(F)$ asks for a factorisation of the complete graph into 2-factors isomorphic to F if n is odd, and for a factorisation of the complete graph into 2-factors isomorphic to F and a single 1-factor if n is even. The problem has been studied extensively, yet a complete solution remains out of reach. In this talk I will survey results on this 50-year-old problem. (Friday 9:00)

Counting regular graphs and not-so-regular graphs

Brendan McKay*, Mikhail Isaev

*Australian National University

The asymptotic number of regular graphs has been the subject of study since Read solved the case d=3 in his 1958 PhD thesis. This was extended to bounded d in 1978, by Wormald and independently by Bender and Canfield. Two years later Bollobás extended this to $d=O(\sqrt{n})$. In 1991, McKay raised the bound to $d=o(n^{1/3})$ and this was raised again to $d=o(n^{1/2})$ by McKay and Wormald in 1991. Meanwhile, McKay and Wormald used complex-analytic methods in 1990 to show that the same asymptotic formula holds for $d \ge n/\log n$. The gap between these two domains remained open for more than a quarter-century, until Liebenau and Wormald closed it with an elegant argument in 2017.

Most of the mentioned results considered not only regular graphs, but graphs with degree sequences having some some limited amount of irregularity. The theorem of Liebenau and Wormald is wide enough to easily include the degree sequences of almost all graphs. In the case where the vertex degrees are linear in the number of vertices, an even wider range was achieved by Barvinok and Hartigan in 2013.

In this talk (joint work with Mikhail Isaev) we describe an extension of the complex-analytic approach that works for average degree at least n^c for any c > 0, and allows a large amount of variation of degrees as well as specification of a possibly large forbidden subgraph. It is based on a bound of Isaev on the truncation error for the cumulant generating function of a complex random variable. In the case of regular graphs, we show that the method leads to an asymptotic expansion and determine the first few coefficients. (Monday 10:50)

Graphs of large chromatic number

Maria Chudnovsky, <u>Alex Scott</u>*, Paul Seymour and Sophie Spirkl.

* Oxford University

Let G be a graph with large chromatic number. What induced subgraphs must it contain? It may contain a large complete subgraph, but what can we say if this is not the case? We will survey recent work on this topic.

(Tuesday 10:50)

Gyárfás-Sumner meets Erdős-Hajnal

Paul Seymour*

*Princeton University

The Gyárfás-Sumner conjecture says that every graph with huge (enough) chromatic number and bounded clique number contains any given tree as an induced subgraph. The Erdős-Hajnal conjecture says that for every graph H, all graphs not containing H as an induced subgraph have a clique or stable set of polynomial size. This talk is about a third problem related to both of these, the following.

Say an n-vertex graph is "c-coherent" if every vertex has degree < cn, and every two disjoint vertex subsets of size at least cn have an edge between them. To prove a given graph H satisfies the Erdős-Hajnal conjecture, it is enough to prove that H satisfies the conjecture in all c-coherent graphs and their complements, for c > 0 as small as we like. But for some graphs H, all c-coherent graphs contain H if c is small enough, so half of the task is done for free.

Which graphs H have this property? Paths do (a theorem of Bousquet, Lagoutte, and Thomassé), and non-forests don't. Perhaps all forests do? (This conjecture is due to Liebenau and Pilipczuk.) More generally, what can we say about induced subgraphs in c-coherent graphs with c very small? For instance:

- If G is c-coherent for c small enough then c contains any given subdivided caterpillar as an induced subgraph (joint with Liebenau, Pilipczuk and Spirkl).
- If G is c-coherent for c small enough then it contains a subdivision of any given graph as an induced subgraph (joint with Chudnovsky, Scott and Spirkl).

There are also some nice parallels between this question and the Gyárfás-Sumner question. (Wednesday 9:00)

On the eigenvectors of random regular graphs and random matrices

Balázs Szegedy*

*Rényi Institute, Hungary

Random regular graphs are fascinating and mysterious objects. Due to extensive work by many researchers in the past decades their literature is rather rich. Despite of this, many fundamental problems in the area are still unsolved. A major open question is a general structural convergence of random d regular graphs on a growing number of vertices. It is widely believed that most reasonable structural parameters converge suggesting that in the limit there is a unique "Platonic" object for every d carrying rich information on the global structure of these graphs. Currently there is not even a conjecture for what this infinite structure could be. This talk tells the story of a promising new approach to these questions that led to a characterisation of almost eigenvectors. Our set of tools include ergodic theory, information theory and graph limit theory. The same methods in a different form can be applied to general random matrices. We report on recent results in this direction too. (Saturday 9:30)

Expanders in finite fields

Le Anh Vinh*

* Vietnam National University

Let \mathbb{F} be a field and E be a finite subset of \mathbb{F}^d , the d-dimensional vector space over the field \mathbb{F} . Given a function $f: \mathbb{F}^d \to \mathbb{F}$, define

$$f(E) = \{ f(x) : x \in E \},$$

the image of f under the subset E. We say that f is a d-variable expander with expansion index ϵ if $|f(E)| \ge C_{\epsilon} |E|^{1/d+\epsilon}$ for every subset E, possibly under some general density or structural assumptions on E.

In this talk, we present a number of results on whether certain polynomials have the expander properties. These results are based on a point-plane incidence bound of Rudnev. (Tuesday 9:00)

Public Lecture

Celebrating the centenary of mathematician Bill Tutte
7pm Tuesday 5th December
South 1 lecture theatre, 43 Rainforest Walk, Monash Clayton

Making links and breaking codes

Graham Farr*

*Monash University

William (Bill) Tutte (1917-2002) became a research mathematician while still an undergraduate at Cambridge in the late 1930s, broke the toughest Nazi codes while at Bletchley Park in the Second World War, and became one of the greatest mathematicians of the 20th century. His wartime work sparked the secret construction of Colossus, one of the first-ever computers, and saved countless lives. After the war, he led the development of the mathematics of networks, known as Graph Theory. His work was usually inspired by pure curiosity or entertaining puzzles, but has been applied in domains as diverse as electrical circuits, statistical physics and information visualisation. This talk tells the story of Tutte's life, mathematics and code-breaking to a broad audience.

Abstracts of contributed talks

Nowhere-zero 3-flows in Cayley graphs on certain finite groups

Milad Ahanjideh* and A. Iranmanesh

* Tarbiat Modares University

One of the famous conjectures about flows in graphs is due to Tutte that asserts every bridgeless graph with no 3-edge cut admits a nowhere-zero 3-flow. This conjecture has been studied for several classes of graphs. It is interesting to investigate the validity of the above conjecture for Cayley graphs. It has been proved that a Cayley graph on abelian groups, nilpotent groups, generalized dihedral groups and generalized quaternion groups admits a nowhere-zero 3-flow. In this talk, as a step towards this, we verify the validity of the above conjecture for cayley graphs on some solvable groups with particular normal subgroups. For example, among other result, we prove that if L is a nilpotent group, then for every generalized dicyclic group $\mathrm{Dic}(H,y)$, the Cayley graph of valency at least 4 on $\mathrm{Dic}(H,y) \times L$ admits a 3-a nowhere-zero 3-flow. These results somehow extend the above results.

(Thursday 11:50)

Wilf-collapse in permutation classes

<u>Michael Albert</u>* and Vít Jelínek and Michal Opler

* University of Otago

A permutation class is a combinatorial class consisting of some finite permutations (thought of as sequences of integers) closed under a natural substructure relation. Two such classes are said to be Wilf-equivalent if there is a size-preserving bijection between them. Given a class \mathcal{C} and a permutation $\pi \in \mathcal{C}$ we consider the subclasses \mathcal{C}_{π} consisting of those permutations in \mathcal{C} that do not contain π . Say $\pi \sim_{\mathrm{WE}} \tau$ if \mathcal{C}_{π} and \mathcal{C}_{τ} are Wilf-equivalent. Let c_n denote the number of permutations in \mathcal{C} of size n, and let w_n denote the number of equivalence classes of \sim_{WE} on permutations in \mathcal{C} of size n. A priori, the only obvious bound we have is that $w_n \leq c_n$. However, it is frequently the case that $w_n = o(c_n)$ in which case we say that \mathcal{C} exhibits a Wilf-collapse. We give some sufficient conditions for Wilf-collapse which apply to a wide variety of permutation classes. (Wednesday 10:00)

Enumeration of sparse uniform hypergraphs with forbidden edges

<u>Haya Aldosari</u>* and Catherine Greenhill

* University of New South Wales

Let X be a hypergraph on vertex set V. We prove an asymptotic formula for the number of simple uniform hypergraphs on V with given degree sequence which contain no edges from X. Our proof uses the switching method on a set of uniform hypergraphs chosen at random. We also mention some applications of the formula. (Tuesday 12:15)

On (1,2)-Invariant Graphs

 $\underline{{\sf Brian\ Alspach}}^*,$ Afsaneh Khodadad
pour and Donald L. Kreher * University of Newcastle

A trivalent vertex-transitive graph X is (0,1)-invariant if it can be decomposed into a 2-factor and a 1-factor such that $\operatorname{Aut}(X)$ preserves the edge partition. We discuss our initial results on looking at the classification of (1,2)-invariant graphs. (Monday 11:50)

Subtrees and independent subsets in unicyclic graphs and unicyclic graphs with fixed segment sequence

<u>Eric Ould Dadah Andriantiana</u>*, Stephan Wagner and Hua Wang
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In the study of topological indices, two negative correlations are well known: that between the number of subtrees and the Wiener index (sum of distances) and that between the Merrifield-Simmons index (number of independent vertex subsets) and the Hosoya index (number of independent edge subsets). That is, among a certain class of graphs, the extremal graphs that maximize one index usually minimize the other, and vice versa. Our study of the numbers of subtrees in unicyclic graphs and unicyclic graphs with a given girth, further confirms its opposite behavior to the Wiener index by comparing with known results on the Wiener index. But when we considered the unicyclic graphs with a given segment sequence, we found that the extremal graph with the maximum number of subtrees, is not always the graph that has the minimum Wiener index. We also identify the extremal structures that maximize the number of independent vertex subsets among unicyclic graphs with a given segment sequence, and show that it is not always extremal with respect to the number of independent edge subsets. These results may be the first examples where the negative correlation failed in the extremal cases between these two pairs of indices. (Thursday 5:30)

Distance Magic Labelings of Distance-Regular Graphs

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Let G be a graph with order n and diameter d. Let $D \subseteq \{0, 1, 2, ..., d\}$ and $N_D(v) = \{u \in V(G) \mid d(u, v) \in D\}$, where $v \in V(G)$. A bijection $l : V(G) \longrightarrow \{1, 2, ..., n\}$ is called D-distance magic labeling of G if there is a nonnegative integer k such that $\sum_{N_D(v)} l(u) = k$ for every $v \in V(G)$. If $D = \{1\}$, D-distance magic labeling is called distance magic labeling.

A distance-regular graph is a regular connected graph with degree k and diameter d, for which following holds. There are natural numbers

$$b_0 = k, b_1, ..., b_{d-1}, c_1 = 1, c_2, ..., c_d,$$

such that for each pair (u, v) of vertices satisfying d(u, v) = j we have

- (1) the number of vertices in $G_{j-1}(v)$ adjacent to u is c_j $(1 \le j \le d)$;
- (2) the number of vertices in $G_{j+1}(v)$ adjacent to u is b_j $(0 \le j \le d-1)$.

The array $\{k, b_1, ..., b_{d-1}; 1, c_2, ..., c_d\}$ is the intersection array of G.

We will present distance magic labelings of distance-regular graphs of diameter 2 (strongly regular graphs) by utilizing eigen values of said graphs. In addition, we will study distance magic labelings of hypercubes, line graphs of complete graphs, and distance-regular graphs of diameter 3. (Friday 10:50)

(K_2, H) -Magic labelings of Graphs

Yeva Fadhilah Ashari*, Rinovia Simanjuntak, and A. N. M. Salman

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Let (H_1, H_2) be two simple and finite graphs. A graph G admits an (H_1, H_2) -covering if every edge in E(G) belongs to a subgraph of G isomorphic to H_1 or H_2 . Suppose G admits an (H_1, H_2) -covering. G is said to be (H_1, H_2) -magic if there exists a bijective function f from $V(G) \cup E(G)$ to $\{1, 2, \ldots, |V(G)| + |E(G)|\}$ and two fixed integers k_1 and k_2 such that $\sum_{v \in V'} f(v) + \sum_{e \in E'} f(e) = k_1$ for every subgraph H' = (V', E') isomorphic to H_1 and $\sum_{v \in V''} f(v) + \sum_{e \in E''} f(e) = k_2$ for every subgraph H'' = (V'', E'') isomorphic to H_2 . Furthermore, G is (H_1, H_2) -supermagic if $f(V(G)) = \{1, 2, \ldots, |V(G)|\}$.

In this paper, we provide neccessary conditions for (K_2, H) -magic graphs where H is isomorphic to a cycle, star, or path. Utilising these necessary conditions, we classify some classes of graphs that are (K_2, H) -magic. (Friday 11:15)

Approaching the Moore bound for diameter 3 by Cayley graphs

Martin Bachratý*, Jana Šiagiová and Jozef Širáň

* University of Auckland

The largest order n(d,k) of a graph of maximum degree d and diameter k cannot exceed the Moore bound, which has the form $M(d,k)=d^k-O(d^{k-1})$ for $d\to\infty$ and any fixed k. Known results in finite geometries on generalised (k+1)-gons imply, for k=2,3,5, the existence of an infinite sequence of values of d such that $n(d,k)=d^k-o(d^k)$. This shows that for k=2,3,5 the Moore bound can be asymptotically approached in the sense that $n(d,k)/M(d,k)\to 1$ as $d\to\infty$; moreover, no such result is known for any other value of $k\geqslant 2$. The corresponding graphs are, however, far from vertex-transitive, and there appears to be no obvious way to extend them to vertex-transitive graphs giving the same type of asymptotic result.

By a detailed analysis of regular orbits of suitable groups of automorphisms of graphs arising from polarity quotients of incidence graphs of generalised quadrangles with polarity, we prove that for an infinite set of values of d there exist Cayley graphs of degree d, diameter 3, and order $d^3 - O(d^{2.5})$. The Moore bound for diameter 3 can thus be asymptotically approached by Cayley graphs. Previously, this was known to be true only for diameter 2. (Monday 12:15)

On the metric dimension of incidence graphs

Robert Bailey*

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The metric dimension of a graph Γ is the least number of vertices with the property that any vertex of the graph can be identified by its list of distances to the chosen few. In this talk, we consider the incidence graphs of symmetric designs (which are precisely the bipartite distance-regular graphs of diameter 3) and of symmetric transversal designs (which are precisely the bipartite, antipodal distance-regular graphs of diameter 4). Using the probabilistic method (adapting an approach used by Babai for strongly regular graphs), we are able to obtain upper bounds on the metric dimension of these graphs, which in certain cases is—asymptotically—the best possible. (Thursday 10:00)

Cages arising from generalised polygons.

John Bamberg*, Anurag Bishnoi, Gordon Royle

* The University of Western Australia

The cage problem asks for the least number c(k, g) of vertices of a graph with a given degree k and girth g. Graphs that attain the minimum number of vertices are called cages. The only known non-trivial infinite families of cages are the generalised polygons, and so

$$c(k,g) = 2((k-1)^{g/2-1} + (k-1)^{g/2-2} + \dots + 1)$$

for $g \in \{6, 8, 12\}$, whenever k-1 is a prime power. We improve the known upper bounds on c(k, 8) and c(k, 12) for infinitely many values of k by constructing new small regular induced subgraphs of known generalised quadrangles and hexagons. We also give a lower bound on the smallest number of vertices in a regular induced subgraph of a generalised polygon, using the *Expander Mixing Lemma*. (Friday 4:40)

Reductions of 3-connected graphs with minimum degree at least four

Sheng Bau*

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We show that if G is a 3-connected graph of minimum degree at least 4 and with $|V(G)| \ge 7$ then one of the following is true: (1) G has an edge e such that G/e is a 3-connected graph of minimum degree at least 4; (2) G has two edges uv and xy with $ux, vy, vx \in E(G)$ such that the graph G/uv/xy obtained by contraction of edges uv and xy in G is a 3-connected graph of minimum degree at least 4; (3) G has a vertex x with $N(x) = \{x_1, x_2, x_3, x_4\}$ and $x_1x_2, x_3x_4 \in E(G)$ such that the graph $(G-x)/x_1x_2/x_3x_4$ obtained by contraction of edges x_1x_2 and x_3x_4 in G-x is a 3-connected graph of minimum degree at least 4.

Each of the three reductions is necessary: there exists an infinite family of 3-connected graphs of minimum degree not less than 4 such that only one of the three reductions may be performed for the members of the family and not the two other reductions. (Tuesday 12:15)

Calculating Connected Components in Huge Graphs by Randomised Contraction

 $\underline{\text{Harald B\"{o}geholz}}^*$, Michael Brand and Radu-Alexandru Todor * $Monash\ University$

We present an algorithm for calculating the connected components of a graph $G = \langle V, E \rangle$ that lends itself to an efficient implementation in a relational database. It is a linear-space randomised algorithm, always terminating with the correct answer but subject to a stochastic running time, terminating after $O(\log |V|)$ steps in expectation. An implementation in a Massively Parallel Processing (MPP) database scales up to huge graphs that are too big to fit into a single machine, making it a practical solution for Big Data analytics.

The algorithm works by contracting the graph to a set of representatives, preserving connectivity, until a set of isolated vertices remains. To analyse its complexity we prove the following theorem: in a directed graph $G = \langle V, E \rangle$, let $N^+(v)$ be the out-neighbourhood of a vertex v and assume all vertices have nonzero out-degree. Given a uniformly-chosen random ordering of the vertices, choose the representative of a vertex v to be $r(v) = \min N^+(v) \cup \{v\}$ under this ordering. We show that in expectation $|\{r(v): v \in V\}|$ is at most $\frac{2}{3}|V|$. This is a tight bound. (Wednesday 10:25)

Detachable pairs in 3-connected matroids

Nick Brettell*, Geoff Whittle and Alan Williams

* Victoria University of Wellington

Let M be a 3-connected matroid with a 3-connected minor N. We say that a pair of elements $\{x,y\}$ in M is N-detachable if either $M/\{x,y\}$ or $M\setminus\{x,y\}$ is 3-connected and has an N-minor. In this talk, we present a result characterising when M has an N-detachable pair. Namely, when |E(M)| - |E(N)| is sufficiently large, either M has an N-detachable pair, the matroid obtained from M by a single Δ -Y or Y- Δ exchange has an N-detachable pair, or M can be obtained by gluing a spike on to N. This work is motivated by recent proofs of excluded-minor characterisations of matroids representable over a certain field or fields. We also touch on the analogous result for 3-connected graphs, implied by this result. (Thursday 4:15)

Lessons from the Non-Existence of a 2-Resolvable BIBD(10,15,6,4,2)

Diana Combe and Yudhistira A. Bunjamin*
* UNSW Sydney

A Balanced Incomplete Block Design (BIBD) is α -resolvable if its block set (or multiset) can be partitioned into α -resolution classes such that each variety is replicated α times among the blocks in each α -resolution class. The question of the existence of a 2-resolvable BIBD(10,15,6,4,2) is motivated by Kadowaki and Kageyama's Conjecture (2003) because its existence would provide a counterexample to the conjecture. There are a number of very different proofs which show that a 2-resolvable BIBD(10,15,6,4,2) does not exist. This talk will compare and contrast four of these proof methods. (Thursday 4:40)

Balanced diagonals in frequency squares

Nicholas Cavenagh* and Adam Mammoliti

* University of Waikato

Anne Penfold Street Memorial Session

Let F be a $m\lambda \times m\lambda$ frequency square in which each entry from $\{1, 2, ..., m\}$ occurs λ times per row and λ times per column. We study when F contains a balanced diagonal; i.e. a diagonal in which each entry occurs λ times. We give necessary and sufficient conditions for the existence of a balanced diagonal when $m \in \{2, 3\}$. We make a conjecture for arbitrary m, generalizing Ryser's conjecture that every Latin square of odd order contains a transversal. (Friday 11:15)

On degree sum conditions for 2-factors with a prescribed number of cycles

Shuya Chiba*

*Kumamoto University

In this study, we consider only finite simple graphs. A 2-factor of a graph is a spanning subgraph in which every component is a cycle. For a vertex subset X of a graph G, let $\Delta_2(X)$ be the maximum degree sum of two distinct vertices of X. We prove the following result: Let k, m be positive integers, and let G be an m-connected graph of order $n \ge 5k - 2$. If $\Delta_2(X) \ge n$ for every independent set X of size $\lceil m/k \rceil + 1$ in G, then G has a 2-factor with exactly k cycles. This is a common generalization of the results obtained by Brandt, Chen, Faudree, Gould, Lesniak (J. Graph Theory, 1997) and Yamashita (Discrete Math., 2008), respectively. (Tuesday 3:00)

Online Colouring Problems in Overlap Graphs and their Complements

Marc Demange* and Martin Olsen

*RMIT University, Australia and BTECH, Aarhus University, Denmark

We consider an online version of different colouring problems in overlap graphs, motivated by some stacking problems. The instance is a system of time intervals presented in non decreasing order of the left endpoint. We consider the usual colouring problem as well as b-bounded colouring (colour class have a maximum capacity b) and the same problems in the complement graph. We also consider the case where at most b intervals of the same colour can intersect. For all these versions we obtain a $O(\log \frac{L}{\ell})$ -competitive online algorithm, where L, ℓ respectively denote the maximum and minimum interval lengths. The best known competitive ratio for the usual colouring was $O(\frac{L}{\ell})$. Our method is based on a partition of the overlap graph into permutation graphs, leading to a competitive-preserving reduction of the problem in overlap graphs to the same problem in permutation graphs. From the hardness point of view now, we show a lower bound of $O(\log \frac{L}{\ell}/\log\log \frac{L}{\ell})$ for the competitive ratio. (Tuesday 3:50)

A quick survey on coclass graphs

Heiko Dietrich*

*Monash University

Leedham-Green & Newman defined the coclass of a finite p-group of order p^n and nilpotency class c as r = n - c. The investigation of the p-groups of a fixed coclass led to deep results in p-group theory (see the book of Leedham-Green & McKay), to applications (e.g. the investigation of Schur multipliers or automorphism groups of p-groups), and to generalisations to other algebraic objects (e.g. algebras or semigroups). In the last decade, the focus in coclass theory is on the investigation of the $coclass\ graph\ \mathcal{G}(p,r)$ associated with the finite p-groups of coclass r: the vertices of this graph are (isomorphism type representatives of) the finite p-groups of coclass r, and there is an edge between two groups G and H if and only if G is isomorphic to $H/\gamma(H)$ where $\gamma(H)$ is the last non-trivial term in the lower central series of H. The graph $\mathcal{G}(p,r)$ is infinite and rich in structure. Indeed, it is a central conjecture that it can be described completely by a finite subgraph and several "periodicity patterns".

The aim of this talk is to give a survey on the known periodicity results for the graph $\mathcal{G}(p,r)$, some outstanding problems, and some recent new results for the graph $\mathcal{G}(p,1)$. The latter are joint work with Bettina Eick and Tobias Moede, respectively. (Friday 11:15)

Efficient Security Repairing in Multi-Cloud Storage using Latin Square Autotopism Secret Sharing

<u>Kaiyue Duan</u>*, Yan Kong, Jinjin Sun, Rebecca J. Stones, Lu Shen, Gang Wang, Xiaoguang Liu and Ming Su

*Nankai University

For secure cloud storage, users can upload encrypted data to multiple remote cloud servers, but there is a risk that users might leak their cryptographic keys. When this occurs, we perform *security repairing*, which requires replacing the cryptographic key(s) and possibly modifying the uploaded ciphertext. In the usual scheme, All-or-Nothing, security repairing is time-consuming because we modify the ciphertext which are at least as large as the original data.

To achieve efficient security repairing, we present a data-dispersion scheme based on a Latin-square-autotopism secret sharing scheme. In this scheme, to perform security repairing, we need only update the secret shares (i.e., the cryptographic keys) and do not need to re-encrypt the uploaded data. We call it the Latin-Square Reed-Solomon (LS-RS) scheme, and it is an adaptation of the All-or-Nothing Reed-Solomon (AONT-RS) multi-cloud system.

We implement and experimentally test this scheme in a prototype multi-cloud storage system. Experiments indicate the proposed scheme is up to 25 times faster than AONT-RS at security repairing.

(Tuesday 3:50)

On the two-colour Rado number for $\sum_{i=1}^{m} a_i x_i = c$

Ishan Arora, <u>Srashti Dwivedi</u>* and Amitabha Tripathi

*Indian Institute of Technology, Delhi

Let a_1, \ldots, a_m be nonzero integers, $c \in \mathbb{Z}$ and $r \geqslant 2$. The Rado number for the equation

$$\sum_{i=1}^{m} a_i x_i = c$$

in r colours is the least positive integer N such that any r-colouring of the integers in the interval [1, N] admits a monochromatic solution to the given equation. We determine exact values whenever possible, and upper and lower bounds otherwise, for the Rado numbers when the set $\{a_1, \ldots, a_{m-1}\}$ is 2-distributable or 3-distributable, $a_m = -1$, and r = 2. This generalizes previous works by Schaal and his co-authors. (Thursday 12:15)

Toughness and treewidth

Mark Ellingham*, Songling Shan, Dong Ye and Xiaoya Zha

* Vanderbilt University, USA

We discuss two results involving toughness and treewidth. First, Jackson and Wormald conjectured that for $k \ge 2$ every $\frac{1}{k-1}$ -tough graph has a spanning closed walk using every vertex at most k times. We show that this is true for graphs of treewidth at most 2, or equivalently for K_4 -minor-free graphs. In fact, we prove the stronger result that for $k \ge 2$ every $\frac{1}{k-1}$ -tough graph of treewidth at most 2 has a spanning tree of maximum degree at most k. Second, computing toughness is NP-hard for general graphs. We show that toughness, or the truth of certain conditions related to toughness, can be determined in polynomial time for graphs of bounded treewidth. (Saturday 11:25)

Graph embedding and the complexity of the DNA reporter strand problem

Joanna Ellis-Monaghan* and Mark Ellingham
*Saint Michael's College

In 2009, Jonoska, Seeman, and Wu showed that every graph admits a route for a DNA scaffolding strand, that is, a closed walk covering every edge either one or two times, in opposite directions if two times. This corresponds to showing that every graph has an orientable embedding with at least one face that is incident with every edge. In the context of the original application, the desired object is such a closed walk of minimum length. Here we give a very short proof of the original result, but more critically, prove that finding a shortest length solution is NP-Hard, even in the special case of 3-regular, 3-connected, planar graphs. Independent of the motivating application, this problem opens a new direction in the study of graph embeddings, and we suggest several new problems emerging from it. (Friday 3:50)

Firefighter Problem on Trees

<u>David Ellison</u>*, Pierre Coupechoux, Marc Demange and Bertrand Jouve *RMIT University, School of Science

In the Firefighter game, a fire spreads through a graph while a player chooses which vertices to protect in order to contain it. An instance is defined by a graph, a vertex which is initially on fire and a sequence, called the firefighter sequence, which determines how many vertices can be protected at each turn. During a turn, the firefighter chooses which vertices to protect, then the fire spreads to all adjacent unprotected vertices. For finite graphs, the objective is to save as many vertices as possible, while for infinite graphs, the objective is to contain the fire. In the online version of the Firefighter problem, the firefighter sequence is revealed over time: the player finds out at each turn how many firefighters he can use. In the case of infinite trees, it seems natural to think that in order to contain the fire, the firefighter sequence must grow fast enough, compared with the number of vertices per level of the tree. Indeed, criteria for deciding whether or not the fire is containable can be derived by comparing the asymptotic behaviours of those sequences, in both the online and offline cases.

(Friday 10:00)

Re-embedding structures of 3-connected 3-regular planar graphs into non-spherical surfaces

Kengo Enami*

* Yokohama National University

Two embeddings f_1 and $f_2: G \to F^2$ of a graph G on a closed surface F^2 are equivalent if there exists a homeomorphism $h: F^2 \to F^2$ with $hf_1 = f_2$. A graph G is uniquely embeddable on F^2 up to equivalence if all embeddings of G on F^2 are equivalent. It is well-known that every 3-connected planar graph is uniquely embeddable on the sphere but it is not uniquely embeddable on any surface other than the sphere. In this situation, two natural questions will arise; how many inequivalent embeddings does a 3-connected planar graph have and what kind of mechanisms generate these embeddings?

We shall focus on a 3-connected 3-regular planar graph and classify structures of its embeddings on the torus, the projective plane and the Klein bottle. This enables us to determine whether a 3-connected 3-regular graph embedded on the one of these surfaces is planar or not. Moreover, we can establish the following two algorithms. An enumeration algorithm is said to have *polynomial delay* if the maximum computation time between an output and the next is polynomial in the input size.

- A polynomial-time algorithm for outputting the total number of inequivalent embeddings of a given 3-connected 3-regular planar graph on the torus, the projective plane or the Klein bottle
- A polynomial delay algorithm for enumerating all the inequivalent embeddings of a given 3-connected 3-regular planar graph on the torus, the projective plane or the Klein bottle

(Thursday 5:05)

Hamiltonian prisms under toughness conditions

Mou Gao*

*Dalian University of Technology

In 1973, Chvátal posed a famous conjecture, claiming that there exists a constant t_0 such that every t_0 -tough graph is Hamiltonian . In recent decades, researchers found many special classes of graphs for which Chvátal's conjecture is true. Moreover, researchers are also interested in many kinds of analogies of Hamiltonian cycles, such as k-walks, Hamiltonian-prisms, and 2-factors. A k-walk in a graph G is a closed walk visiting each vertex of G at least once but at most k times. The prism over a graph G is the Cartesian product $G \times K_2$ of G with the complete graph K_2 . If $G \times K_2$ is Hamiltonian, then we say G is prism-Hamiltonian, and we call $G \times K_2$ the Hamilton-prism of G. It is not difficult to prove that being prism-Hamiltonian is a property stronger than admitting 2-walk but weaker than being Hamiltonian. For k-walks in graphs, there is also a well-known open conjecture, which is posed by Jackson and Wormald, saying that every $\frac{1}{k-1}$ -tough graph admits a k-walk.

Recently, researchers have confirmed Jackson-Wormald's Conjecture for $2K_2$ -free graphs and K_4 -minor-free graphs.

A graph G is called k-chordal, if for any cycle C in G with length $|C| \ge k$, C has a chord in G. Usually, we call a 3-chordal graph a chordal graph for convenience. It is proved, in 1998, that Chvátal's Conjecture is true for chordal graphs, with the toughness bound 18. In 2016, this bound was improved into 10. However, Jackson-Wormald's Conjecture is still open for chordal graphs. Recently, we are interested in 4-chordal graphs. (Monday 10:00)

2-arc-transitive digraphs

Michael Giudici*

* The University of Western Australia

An s-arc in a digraph Γ is a sequence v_0, v_1, \ldots, v_s of vertices such that for each i the pair (v_i, v_{i+1}) is an arc of Γ . There are several important differences between the study of s-arc-transitive graphs and s-arc transitive digraphs. For example, there are no 8-arc-transitive graphs of valency at least 8, while for every positive integer s there are infinitely many digraphs of valency at least three that are s-arc-transitive but not (s+1)-arc transitive. In this talk I will discuss recent work with Cai Heng Li and Binzhou Xia on characterising vertex-primitive 2-arc-transitive digraphs. (Friday 10:50)

Norman involutions and tensor products of unipotent Jordan blocks

Stephen Glasby*, Cheryl Praeger, Binzhou Xia

*Centre for Mathematics of Symmetry and Computation, University of Western Australia

The tensor product of two unipotent Jordan blocks is conjugate to a direct sum of unipotent Jordan blocks. Given integers $r \leq s$, we write $\mathrm{JCF}(J_r \otimes J_s) = \bigoplus_{i \geq 1} J_{\lambda_i}$ where JCF denotes Jordan canonical form, and the subscripts denote dimensions. The partition $\lambda = (\lambda_1, \lambda_2, \ldots)$ of rs depends on r, s and the field F. We write λ as $\lambda(r, s, p)$ when $\mathrm{char}(F) = p > 0$. This talk studies properties of λ and notes that it sheds light on the action of p-groups of matrices in characteristic p, and whether matrix groups are \otimes -indecomposable.

We study certain permutations $\pi(\lambda)$ of $\{1, 2, ..., r\}$ as a proxy to studying the partitions λ . These permutations were defined by Norman in 1995. We give necessary and sufficient conditions for $\pi(\lambda)$ to be trivial, building on work of M. J. Barry. We show that when $\pi(\lambda)$ is nontrivial, it is an involution involving reversals, and we conjecture that the group G(r,p) generated by all the permutations $\pi(r,s,p)$ "factors" as a wreath product corresponding to the factorisation r=ab as a product of its p-part a and a-part b: precisely a-part a-part a-precisely a-part a-part

References

[1] Barry, Michael J. J. On a question of Glasby, Praeger, and Xia, Comm. Algebra 43 (10), 4231–4246.

(Thursday 11:50)

On a generalisation of the Fibonacci word to an infinite alphabet

Amy Glen*, Jamie Simpson, and W. F. Smyth

*Murdoch University

The well-known Fibonacci word F over the binary alphabet $\{0,1\}$ is the fixed point of the morphism $\psi: 0 \mapsto 01, 1 \mapsto 0$ given by

$$F = \lim_{n \to \infty} \psi^n(0) = 010010100100101001010 \cdots$$

Recently Zhang, Wen, and Wu [Electronic J. Combinatorics 24–2 (2017) #P2.52] introduced an interesting generalisation of this infinite word. As an alphabet they used the non-negative integers, and as a morphism they used $\phi: (2i) \mapsto (2i) \cdot (2i+1), (2i+1) \mapsto (2i+2)$ to give the infinite word beginning

$$W = 012232342344523445456 \cdots$$

Some of the properties of the Fibonacci word F have parallels with those of W. For example, if we reduce the elements of the W modulo 2 we obtain F.

In this talk we consider counting the number of occurrences of squares, palindromes, and Lyndon factors in the finite words $W_n := \phi^n(0)$, whose lengths are Fibonacci numbers (analogous to the so-called *finite Fibonacci words* $F_n := \psi^n(0)$). (Monday 3:00)

Star decompositions of random regular graphs

 $\frac{\text{Catherine Greenhill}^*}{*UNSW} \text{ Sydney}$

Anne Penfold Street Memorial Session

In 2016, Delcourt and Postle proved that under the necessary divisibility conditions, with probability which tends to 1, a random 4-regular graph has an edge-decomposition into disjoint copies of $K_{1,3}$. Their proof used the small subgraph conditioning method of Robinson and Wormald.

A result of Lovász, Thomassen, Wang and Zhu (2013) implies that if $3 \le s \le \lceil d/2 \rceil$ then a d-edge-connected graph G has an edge-decomposition into copies of $K_{1,s}$ whenever s divides E(G). Since random d-regular graphs are d-connected (and hence d-edge-connected) with probability which tends to 1, this implies that a random d-regular graph has an edge-decomposition into copies of $K_{1,s}$, with probability which tends to 1, whenever 2s divides dn and $s \le \lceil d/2 \rceil$. Mikhail Isaev and I aim to generalise Delcourt and Postle's result to cover the remaining cases, namely, when 2s > d. This is work in progress: for now, we (believe that) we have proved a generalisation to edge-decompositions of 2r-regular graphs into copies of $K_{1,r+1}$. (Friday 10:50)

k-center in graphs with uncertainty on vertices

M. A. Haddad*, M. Demange and C. Murat

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For a connected graph G = (V, E), the classical k-center problem is to find a set K of k vertices that minimizes the radius which is the maximum distance between any vertex and K. We propose a variant with uncertainty on vertices. This model is inspired by a forest fire management problem. The graph represents the adjacency of zones of a landscape, where each vertex represents a zone. We consider a finite set of fire scenarios with related probabilities. Given a k-center K, its radius may change in some scenarios since some evacuation paths become impracticable. The objective is to find a robust k-center that minimizes the expected value of the radius over all scenarios.

We study this new problem where scenarios are limited to a single burning vertex. First results deal with polynomial methods on paths and cycles, as well as on some classes of trees. (Wednesday 10:00)

Isomorphism classes of difference covering arrays and pseudo-orthogonal Latin squares.

 $\frac{\text{Joanne Hall}^*}{*RMIT \ University}$

We develop connections between difference covering arrays and Skolem sequences and use this connection to show that there are at least two isomorphism classes of difference covering arrays of order congruent to 0 or 2 modulo 8 with three columns. This leads directly to the result that there are at least two paratopism classes of pairs of pseudo-orthogonal Latin squares of order congruent to 0 or 2 modulo 8.

(Tuesday 4:15)

Hamilton Path Decompositions of Complete Multipartite Graphs

Darryn Bryant, <u>Hao Chuien Hang</u>*, Sarada Herke
* University of Queensland

There has been interest in problems concerning decomposition of graphs into Hamilton cycles and into Hamilton paths, for many years. In 1976, Laskar and Auerbach showed that a complete multipartite graph can be decomposed into Hamilton cycles if and only if it is regular of even degree. We prove a corresponding result for decompositions of complete multipartite graphs into Hamilton paths. In particular, we prove that a complete multipartite graph K with N > 1 vertices and M edges can be decomposed into edge-disjoint Hamilton paths if and only if $\frac{m}{n-1}$ is an integer and the maximum degree of K is at most $\frac{2m}{n-1}$. (Monday 12:15)

Designs and Minimal 2-Neighbour-Transitive Codes

Michael Giudici, <u>Daniel Hawtin</u>* and Cheryl Praeger

* University of Western Australia

For the purposes of this talk, a *code* is a subspace of the vector space $V = GF(2)^m$, so that V represents the set of all binary strings of length m. A binary string of length m is the *characteristic* vector of a subset of $M = \{1, \ldots, m\}$, where a 1 in the i-th position denotes inclusion, and 0 exclusion. Identify each subset of M with its corresponding characteristic vector. Let C be a code containing no subset of size less than 5, other than the empty set.

In the above setting, C is 2-neighbour-transitive if and only if its automorphism group contains a subgroup that fixes the zero vector and acts transitively on the set of all 2-subsets of M. It follows from this that if C is 2-neighbour-transitive, then the set of all codewords in C of a fixed size (as a subset) forms a combinatorial object called a 2-design.

I will discuss: generation of minimal 2-neighbour-transitive codes from 2-designs, new bounds on the *minimum distance* (size of the smallest non-empty codeword) of some of these codes, as well as the broader context surrounding this work. (Monday 3:50)

When graphs are too small to be avoided

<u>Kevin Hendrey</u>* and Sergey Norine
*Monash University

Given a graph H, there is a minimum number f(H) such that if the average degree of a graph G is greater than f(H), then G contains H as minor. Over the past 80 years, the value of this extremal function has been precisely determined for various graphs, including infinitely many complete bipartite graphs. However, computing f(H) appears to be very difficult in general for graphs on more than 10 vertices. This talk examines upper bounds on f(H) based on the number of vertices and edges in H. (Tuesday 11:50)

The worst case for k-independence

Nevena Francetić, <u>Sara Herke</u>* and Daniel Horsley

* The University of Queensland

Anne Penfold Street Memorial Session

Let k be a fixed positive integer. A subset S of vertices of a graph is called a k-independent set if the subgraph induced by S has maximum degree less than k. The well-known algorithm MAX repeatedly removes a vertex of maximum degree until the resulting graph has maximum degree less than k. We discuss a procedure that determines, for a given degree sequence D, the smallest size of a k-independent set that can result from any application of MAX to any loopless multigraph with degree sequence D. Furthermore, this analysis of the worst-case behaviour of MAX is sharp. (Friday 11:40)

Comparison of frame potentials of random point configurations on the sphere

Masatake Hirao*

*Aichi Prefectural University

In this talk we deal with determinantal point processes and the jittered sampling on the sphere. The former are used in a fermion model in quantum mechanics and also studied in probability theory, and the latter is one of the famous random sampling method. From the viewpoint of frame potential, we compare these random point configurations with spherical designs, which are one of the non-random "good" point configurations on the sphere. We also discuss random matrices induced by these random configurations if possible. (Tuesday 3:50)

Playing impartial games: from how to win to how to lose

Nhan Bao Ho* and Vladimir Gurvich
*La Trobe University

In a normal impartial game, two players alternately move without skipping. The player who makes the last move wins. Examples include Chess. In the misère version, the player who makes the last move loses. Unlike Chess which is not relevant when playing in misère version, many impartial games turn to be difficult in misère play.

We consider some links between two versions and classify impartial games based on their tameness. (Monday 3:00)

Finite Minkowski planes of type 17 with respect to homotheties

<u>Duy Ho</u>* and Günter Steinke

* *University of Canterbury*

In 1992, Monica Klein described 23 possible types for the classification of Minkowski planes with respect to linearly transitive groups of homotheties. It is known that 15 of these 23 types cannot occur as types of finite Minkowski planes. One case remaining open is that of type 17. It is shown that planes of this type can be represented with (dual) nearfields and furthermore are determined by automorphisms of the multiplicative group of these nearfields. We conjecture that there are no finite Minkowski planes of type 17. In this talk we discuss the validity of the conjecture. (Tuesday 4:40)

Sperner partition systems

Yanxun Chang, Charles Colbourn, <u>Daniel Horsley</u>*, Junling Zhou *Monash University

Anne Penfold Street Memorial Session

An (n, k)-Sperner partition system is a collection \mathcal{P} of partitions of an n-set X such that each partition has k nonempty classes and no class of one partition is a subset of a class of any other. These objects were introduced in 2005 by Meagher, Moura and Stevens. Most of the work on them has focussed on the problem of determining the maximum number of partitions that an (n, k)-Sperner partition system can have. This talk will outline what is already known about this topic and discuss some work in progress, including new upper and lower bounds on this maximum. (Friday 12:05)

Symmetries of Codes in Affine Geometries

John Bamberg, Alice Devillers, Mark Ioppolo*, Cheryl Praeger

* University of Western Australia

In his PhD thesis, Delsarte defined a *code* to be a subset of points in an association scheme. When working in the Hamming scheme, Delsarte's codes correspond to the usual notion of error control codes. However, we may obtain new and interesting combinatorial objects when working in alternative association schemes. In this talk I will discuss the classification of a highly symmetric family of designs called *Delandtsheer designs* which correspond to codes in the Johnson scheme. I will focus on the case where the point set of the design corresponds to the point set of a finite affine geometry AG(n, q). (Tuesday 5:30)

Subgraph counts for dense graphs with specified degrees

Catherine Greenhill, <u>Mikhail Isaev</u>*, Brendan McKay *School of Mathematical Sciences, Monash University

We consider a uniformly chosen random graph G with given degree sequence in the dense range (degrees approximately a constant fraction of the number of vertices). For a given graph H, we find expected numbers of subgraphs and induced subgraphs of G isomorphic to H. Based on results of [2], these problems are reduced to determining of the expectation of certain functions of a random permutation. This is done by applying a general theory (developed in [1]) for the exponential of a martingale. As illustrations, we present formulas for expected numbers of perfect matchings, cycles, independent sets and spanning trees in this random graph model.

- [1] M. Isaev and B.D. McKay. Complex martingales and asymptotic enumeration. *Random Structures and Algorithms* (to appear), e-preprint arXiv:1604.08305.
- [2] B.D. McKay. Subgraphs of dense random graphs with specified degrees. *Combinatorics, Probability and Computing*, 20:413–433, 2011.

Imprimitive symmetric graphs which are not multicovers of their quotients

Bin Jia*

*Mathematical Research Pty Ltd

Let G be a finite X-symmetric graph. I give a sufficient and necessary condition for the existence of a class of finite (X, s)-arc-transitive imprimitive graphs H that are homomorphic to G and are not multicovers of G.

I will present relationships between G and H in regard to algebra, structure, and topology. (Friday 4:15)

On a sufficient condition of a graph with boxicity at most its chromatic number

Akira Kamibeppu*

*Shimane University

The boxicity of a graph G, denoted by box(G), is the minimum nonnegative integer k such that the graph can be isomorphic to the intersection graph of a family of boxes in Euclidean k-space, where a box in Euclidean k-space is the Cartesian product of k closed intervals on the real line. So far it is known that there are some relationships between boxicity and chromatic number. Recently Chandran et al. pointed out that almost all graphs satisfy $box(G) > \chi(G)$, which is based on the probabilistic method, where $\chi(G)$ denotes the chromatic number of a graph G. The family of graphs with $box(G) \leq \chi(G)$ is not narrow. For example, not only simple examples like complete graphs, paths, cycles and their complements but also asteroidal triple free graphs satisfy the inequality $box(G) \leq \chi(G)$.

In this talk, we give a sufficient condition of graphs with $box(G) \leq \chi(G)$. Moreover we present an example of a family of graphs with the condition. Our result also turns out to be a clarification of some graphs with $box(G) = O(\Delta(G))$ that Chandran et al. conjectured for a graph, where $\Delta(G)$ is the maximum degree of a graph G. (Tuesday 5:05)

Spaces of phylogenetic networks

Jonathan Klawitter*

* University of Auckland

Phylogenetic networks are rooted directed acyclic graphs that represent evolutionary relationships between species. Such networks generalise phylogenetic trees since they do not only represent speciation events but also reticulation events like hybridisation or horizontal gene transfer. Distance functions on phylogenetic networks allow to measure similarities and dissimilarities of different hypothesised networks for the same set of species. A common tool to obtain such a distance function is the use of network rearrangement operations that transform one network into another one via local changes. An example of such a rearrangement operation is the graph-theoretic SubNet Prune and Regraft (SNPR) operation, which is a recent generalisation of the popular Subtree Prune and Regraft (SPR) operation on phylogenetic trees. These rearrangement operations induce a space of phylogenetic networks that can be visualised by a graph in which each vertex represents a phylogenetic network and two vertices are joined by an edge precisely if they are one operation aprat. In this talk we discuss shortest paths in the graph that visualises the space induced by SNPR. (Tuesday 10:00)

White's conjecture and related matroid problems

Michał Lasoń*

* University of Bern

When an ideal is defined only by combinatorial means, one expects to have a combinatorial description of its set of generators. An attempt to achieve this description often leads to surprisingly deep combinatorial questions. White's conjecture is an example. It asserts that the toric ideal associated to a matroid is generated by quadratic binomials corresponding to symmetric exchanges. In the combinatorial language this means that if two multisets of bases of a matroid have equal union (as a multiset), then one can pass between them by a sequence of symmetric exchanges between pairs of bases. The conjecture resisted numerous attempts since its formulation in 1980. We will review the progress made in last years, which led to confirmation of the conjecture for high degrees with respect to the rank of a matroid. We will also discuss relations with other intriguing problems on matroids. (Thursday 3:50)

Exact Algorithms via Multivariate Subroutines

Serge Gaspers and Edward J. Lee*
* UNSW Sydney

We consider the family of Φ -Subset problems, where the input consists of an instance I of size N over a universe U_I of size n and the task is to check whether the universe contains a subset with property Φ (e.g., Φ could be the property of being a feedback vertex set for the input graph of size at most k). Our main tool is a simple randomized algorithm which solves Φ -Subset in time $(1+b-\frac{1}{c})^nN^{O(1)}$, provided that there is an algorithm for the Φ -Extension problem with running time $b^{n-|X|}c^kN^{O(1)}$. Here, the input for Φ -Extension is an instance I of size N over a universe U_I of size n, a subset $X \subseteq U_I$, and an integer k, and the task is to check whether there is a set Y with $X \subseteq Y \subseteq U_I$ and $|Y \setminus X| \leqslant k$ with property Φ . We also derandomize this algorithm at the cost of increasing the running time by a subexponential factor in n, and we adapt it to the enumeration setting where we need to enumerate all subsets of the universe with property Φ . This generalizes the results of Fomin et al. [STOC 2016] who proved them for the case b=1. As case studies, we use these results to design faster deterministic algorithms for

- checking whether a graph has a feedback vertex set of size at most k,
- enumerating all minimal feedback vertex sets,
- \bullet enumerating all minimal vertex covers of size at most k, and
- enumerating all minimal 3-hitting sets.

We obtain these results by deriving new $b^{n-|X|}c^kN^{O(1)}$ -time algorithms for the corresponding Φ -EXTENSION problems (or the enumeration variant). In some cases, this is done by simply adapting the analysis of an existing algorithm, in other cases it is done by designing a new algorithm. Our analyses are based on Measure and Conquer, but the value to minimize, $1 + b - \frac{1}{c}$, is unconventional and leads to non-convex optimization problems in the analysis. (Monday 10:00)

Bounding the cop-number of a graph in terms of its genus

Nathan Bowler, Joshua Erde, <u>Florian Lehner</u>*, and Max Pitz $*University\ of\ Warwick$

The cop-and-robber game is a game on a graph played between two players, a set of cops, and a single robber. The rules of the game are as follows: In the first round both the cops and the robber choose starting vertices, in each consecutive even round each cop can move to a neighbouring vertex, in odd rounds, the robber can move to a neighbouring vertex. The game has perfect information, meaning that each player knows the positions of both the cops and the robber at any point in time. The cops win, if after some finite number of steps one of them occupies the same vertex as the robber, otherwise the robber wins.

A graph G is called k-cop win, if for a set of k cops there is a strategy to win the game no matter how the robber plays. The cop number c(G) is the least number k such that G is k-cop win. This notion was first introduced in 1984 by Aigner and Fromme, who showed that the cop number of a planar graph is at most 3. Using similar techniques, Quilliot showed that if G can be embedded on an orientable surface of genus g, then $c(G) \leq 2g + 3$. Schröder later improved this bound to $\frac{3}{2}g + 3$ and conjectured an upper bound of g + 3.

Using a reduction to a topological game played on an orientable surface, we are able to further improve the bound to $\frac{4}{3}g + 3$ which to our knowledge is the first improvement since Schröder first formulated his conjecture in 2001. (Monday 11:50)

Yet another database of strongly regular graphs

Paul Leopardi*

*Bureau of Meteorology, and University of Melbourne

The Cayley graphs of bent Boolean functions are strongly regular graphs [Bernasconi and Codenotti, 1999]. Programs and libraries such as Nauty and Bliss have made it possible to classify large numbers of such strongly regular Cayley graphs by graph isomorphism.

This talk describes work in progress to produce and publish a "big" database of strongly regular Cayley graphs of bent Boolean functions in up to 8 dimensions. (Friday 3:00)

Wilf equivalences in some small permutation classes

Michael Albert and <u>Jinge Li</u>*

* University of Otago

Two classes of combinatorial structures are said to be Wilf-equivalent if they contain the same number of structures of each size. In this work we study Wilf-Equivalences among principal subclasses of the class Av(X) where X consists of exactly two permutations of size 3. Due to symmetry only four cases need to considered: Av(312,123), Av(312,213), Av(312,231), Av(312,321). On each of these classes we determine an equivalence relation directly related to the structure of the permutations which coincides with Wilf-equivalence. (Wednesday 10:25)

Strongly Sequenceable Groups

<u>Georgina Liversidge</u>* and Brian Alspach * *University of Newastle*

Given a Cayley digraph \overrightarrow{X} on a group G with connection S, an orthogonal directed path in \overrightarrow{X} is a directed path \overrightarrow{P} of length |S| with one arc generated by s for each $s \in S$. An analogous definition applies for an orthogonal directed cycle. A group G is said be strongly sequenceable if every connected Cayley digraph on G admits either an orthogonal directed path or an orthogonal directed cycle.

Alspach (2001) conjectured that finite cyclic groups are strongly sequenceable. It has been shown to be true for cyclic groups of order at most 25 and for special connection sets. We investigate the extension of some of the scattered known results. (Tuesday 3:00)

Locating arrays with error correcting ability

Xiao-Nan Lu* and Masakazu Jimbo

* Tokyo University of Science

Locating arrays (LAs) are combinatorial arrays which are introduced for identifying interaction faults and their locations in a component-based system. C. J. Colbourn and D. W. McClary (J Comb. Optim., 15: 17–48, 2008) proposed the notion a (\bar{d}, \bar{t}) -LA for a complex system which contains (at most) d faults, each involving (at most) t interacting factors.

In this talk, by combining the ideas of interaction testing and error-correcting codes, we will introduce a notion of a (\bar{d}, \bar{t}) -LA with error-correcting ability e. In particular, we will give a construction of a $(\bar{1}, \bar{t})$ -LA with error-correcting ability e containing q factors and r levels for each factor, for $q \equiv 1 \pmod{r}$ being a prime power. Furthermore, we will show the asymptotic existence of such arrays, and give some further improvements based on this construction. (Monday 4:40)

On weakly distinguishing graph polynomials

Johann A. Makowsky* and Vsevolod Rakita

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Let P be a graph polynomial. A graph G is P-unique if every graph H with P(G;X) = P(H;X) is isomorphic to G. A graph H is a P-mate of G if P(G;X) = P(H;X) but H is not isomorphic to G. In [No] P-unique graphs are studied for the Tutte polynomial T(G;X,Y), the chromatic polynomial $\chi(G;X)$, the matching polynomial m(G;X) and the characteristic polynomial char(P;X).

A graph polynomial P is almost complete if almost all graphs G are P-unique, and it is weakly distinguishing if almost all graphs G have a P-mate. In [BPR] it is conjectured that almost all graphs are χ -unique, and hence T-unique, in other words, both $\chi(G;X)$ and T(G;X,Y) are almost complete. There are plenty of trivial graph polynomials which are weakly distinguishing, like $X^{|V(G)|}$ or $X^{|E(G)|}$. However, one might expect that the prominent graph polynomials from the literature are not weakly distinguishing. However, here we show that various non-trivial graph polynomials are still weakly distinguishing.

For a vertex $v \in V(G)$ of a graph G let $d_G(v)$ denote the degree of v in G. The degree polynomial Dg(G;X) of a graph G is defined as $Dg(G;X) = \sum_{v \in V(G)} X^{d_G(v)}$. A graph G is Dg-unique, also called in the literature a unigraph, if it is determined by its degree sequence. An updated discussion on how to recognize unigraphs can be found in [BCP].

A simple counting argument gives:

Theorem 1: Almost all graphs G have an Dq-mate.

For a graph G = (V(G), E(G)) and $A \in V(G)$ we denote by G[A] the subgraph of G induced by A. Let \mathcal{E} be the class of edgeless graphs and \mathcal{C} be the class of complete graphs. The independence polynomial of a graph is defined as $Ind(G;X) = \sum_{A \in V(G):G[A] \in \mathcal{E}} X^{|A|}$. The clique polynomial of a graph is defined as $Cl(G;X) = \sum_{A \in V(G):G[A] \in \mathcal{C}} X^{|A|}$. Both were first studied in [HL]. For a more recent survey on the independence polynomial see [LM].

Theorem 2: Almost all graphs G have an Ind-mate and an Cl-mate.

The proofs use estimates for the independence number $\alpha(G)$ and the clique number $\omega(G)$ for random graphs, see [BE,Fr].

A vertex coloring of G with at most k colors is a proper coloring of G such that every pair of colors occurs at most once along an edge. Let $\chi_{harm}(G;k)$ count the number of harmonious colorings of G. It was observed in [MKZ] that $\chi_{harm}(G;k)$ is a polynomial in k.

Theorem 3: Almost all graphs G have an χ_{harm} -mate.

The proof uses results from [DG]. The same holds for an infinite set of less prominent graph polynomials, but the status of P-uniqueness remains open for $T(G; X, Y), \chi(G; X), m(G; X)$ and char(G; X).

References

- [AT] M. Aigner and E. Triesch, Realizability and uniqueness in graphs, Discrete Mathematics, 136 (1994), pp. 3-20
- [BCB] A. Borri, T. Calamoneri and R. Petreschi, Recognition of Unigraphs through Superposition of Graphs, Journal of Graph Algorithms and Applications, 15.3 (2011), pp. 323-343
- [BPR] B. Bollobás and L. Pebody and O. Riordan, Contraction—deletion invariants for graphs, Journal of Combinatorial Theory, Series B, 80.2 (2000), pp. 320-345
- [BG] B. Bollobás and P. Erdős, Cliques in random graphs, Math. Proc. Camb. Phil. Soc. 80 (1976), pp. 419-427
- [DG] E. Drgas-Burchardt and K. Gibek, Harmonious colorings of graphs, Discrete Applied Mathematics, 217 (2017), pp. 175-184
- [Fr] A.M. Frieze, On the independence number of random graphs, Discrete Mathematics 81 (1990), pp 171-175
- [HL] C. Hoede and X. Li, Clique polynomials and independent set polynomials of graphs, Discrete Mathematics, 125 (1994), pp. 219-228
- [LM] V.E. Levit and E. Mandrescu, The independence polynomial of a graph a survey, Proceedings of the 1st International Conference on Algebraic Informatics, Aristotle University of Thessaloniki, Thessaloniki, 2005, pp. 233-254
- [MKZ] J.A. Makowsky and T. Kotek and B. Zilber On Counting Generalized Colorings, in: Model Theoretic Methods in Finite Combinatorics, Contemporary Mathematics, 558 (2011), pp. 207-242
- $[\mathbf{No}]$ M. Noy, On Graphs Determined by Polynomial Invariants, Theoretical Computer Science, 307 (2003), pp. 365-384

(Monday 3:50)

The Erdős-Ko-Rado Theorem, Generalisations and Beyond

Adam Mammoliti*

* UNSW Sydney

Extremal Set Theory is a branch of Extremal Combinatorics where one characterises the maximum size of families of sets with given restrictions placed on them. The Erdős-Ko-Rado Theorem is a classical result in Extremal Set Theory that has, since its discovery, been extensively researched and generalised.

In this talk, I will present an introduction of the Erdős-Ko-Rado Theorem and some of its generalisations. I also will present some of my own results which relate to a particular generalisation of this theorem. Open problems as well as new possible directions of research are also given. (Thursday 10:00)

Covers and partial transversals of Latin squares.

Darcy Best, <u>Trent Marbach</u>*, Rebecca Stones, and Ian Wanless *Monash University

The topic of transversals within Latin squares became of interest through the study of mutually orthogonal Latin squares (MOLS). Along with this connection to MOLS, transversals have been of interest in the literature recently and have had a number of papers study them in their own right. It is well known that some Latin squares contain no transversals, but what is not so clear is how *close* a Latin square must be to containing a transversal. Previously, this problem has been studied by studying partial transversals of Latin squares and observing how large we can make such a partial transversal. In this presentation, we will discuss an alternate substructure of a Latin square that can also be considered to be *close* to a transversal, which we call a cover.

A cover of a Latin square is a subset of entries of the Latin square such that each row, column, and symbol is represented at least once in the set of entries. We will present the results we have found regarding covers of minimum size within a Latin square, which shows a kind of duality between covers and partial transversals. We also will present work on minimal covers, which shows a clear distinctiveness between covers and partial transversals. After this, we will demonstrate a few other results of interest on this topic.

(Tuesday 5:05)

Nonexistence of some linear codes over the field of order four

Hitoshi Kanda and <u>Tatsuya Maruta</u>*

* Osaka Prefecture University

We consider the problem of determining $n_q(k, d)$, the smallest possible length n for which an $[n, k, d]_q$ code of fixed dimension k and minimum distance d over the field of order q exists. This problem is sometimes called the optimal linear codes problem. The optimal linear codes problem for q = 4 is solved for all $k \leq 4$ and for k = 5 for all but 107 values of d. We tackle some open cases for q = 4 and k = 5 using the known extension theorems for quaternary linear codes. (Monday 4:15)

The relations between even block designs as finite groups

Maryam Tale Masouleh*, Ali Iranmanesh, Henk koppelaar

* Tarbiat Modares University

An ordered pair (G, β) is an even block design, if |G| is an even number and (G, β) is an BIBD. An (H, β') -BIBD is called a sub-design of (G, β) -BIBD, if $H \subseteq G$ and $\beta' \subseteq \beta$. In this talk, we will show that every pair of even block design, mutually are embeddable as a sub design. It is done by graph theory methods and some properties of finite groups. (Thursday 4:15)

Cycle structures of certain autoparatopisms of Latin Squares

Mahamendige Jayama Lalani Mendis*

* University of Colombo, Sri Lanka

Let $\theta = (\alpha, \beta, \gamma) \in S_n^3$ and $\sigma = (\theta; \lambda) \in S_n \wr S_3$. Permute rows, columns and symbols of a Latin square L by α, β , and γ to derive a new Latin square L^{θ} . Then another Latin square L^{σ} is obtained permuting triples of L^{θ} by λ . If $L^{\sigma} = L$ then σ is known as an autoparatopism of Latin square L. Stones, Vojtěchovský and Wanless established number of necessary conditions for (α, β, γ) to determine autoparatopisms for $n \leq 17$ when λ is an identity. Mendis and Wanless worked to obtain results to find autoparatopisms for $n \leq 17$ when λ is 2 and 3-cycles. We can observe from known autoparatopisms that most cycle structures of γ represent when the cycle structure of β is d^m , where d, m are positive integers. Accordingly, I found the full picture of cycle structures of γ such that $(\varepsilon, d^m, \gamma; (12))$ is an autoparatopism. (Tuesday 5:30)

Exploring the projective norm graph

Tomas Bayer, <u>Tamás Mészáros</u>*, Lajos Rónyai, Tibor Szabó *Freie Universität Berlin

The projective norm graphs NG(q,t) were introduced by Alon, Rónyai and Szabó as a tight construction for the Turán problem for complete bipartite graphs $K_{t,s}$ when s > (t-1)!. Since their first appearance they have served as useful constructions for other combinatorial problems as well, and many of their interesting properties have been proven. Here we explore them further: we discuss their automorphism group and study their small subgraphs. In particular, we count the 3-degenerate subgraphs and prove that the $K_{4,7}$ -free projective norm graph NG(q,4) does contain a $K_{4,6}$. Some of these results extend the work of Alon and Shikhelman on generalized Turán numbers. (Thursday 12:15)

On combinatorial structures constructed from unitary groups $U(3,q), \ q=3,4,5,7$

Dean Crnković, <u>Vedrana Mikulić Crnković</u>* and Andrea Švob
* *University of Rijeka, Croatia*

The method of constructing 1-designs from a transitive permutation group will be described. By using this method, numerous designs on which the unitary group $U(3,q), q \in \{3,4,5,7\}$, acts transitively were constructed. For example, the Hermitian unital $2-(q^3+1,q+1,1)$, a flag-transitive $2-(q^3+1,q,q-1)$ design, a semi-symmetric $(q^4-q^3+q^2,q^2-q,(1))$ design were obtained. Moreover, the construction of the Desarguesian projective plane $PG(2,q^2)$ from the unitary group $U(3,q), q \in \{3,4,5,7\}$, will be introduced. (Friday 11:40)

Cayley digraphs and Cayley index

<u>Luke Morgan</u>* and Joy Morris and Gabriel Verret

* The University of Western Australia

The Cayley index of a Cayley digraph on a finite group G is the index of (the regular representation of) G in the automorphism group of that digraph. The minimum Cayley index of one is attained by digraphs called DRRs (Digraphical Regular Representations). These are classified by Babai who shows that, apart from five groups, every finite group admits a DRR. That is, apart from five exceptions, every finite group G has a Cayley digraph such that G is the full automorphism group of that digraph. In this talk, we'll consider the question of what might make the Cayley index "large" relative to the number of vertices (= the order of the group). A result of Morris shows that any Cayley digraph on a cyclic p-group (with p an odd prime) has Cayley index super-exponential in p, if there exists another distinct regular subgroup. This result was later generalised to p = 2. So these results say that if a digraph is Cayley for two distinct p-groups, one of which is cyclic, the Cayley index is forced to be "large". In joint work with Morris and Verret, we considered if cyclic p-groups are exceptional in this respect. We found that, in contrast to the previous results, every non-cyclic abelian p-group (p odd) of order at least p^3 admits a Cayley digraph of Cayley index p that admits two distinct regular subgroups (and that this Cayley index is minimal). I'll show how this result works, and give some further questions on this topic. (Monday 4:15)

Purity of projective planar graphs

Raiji Mukae* and Terukazu Sano

*National Institute of Technology, Kisarazu College

A graph class \mathcal{G} is pure if |E(G)| = |E(H)| for all edge-maximal graphs $G, H \in \mathcal{G}$ with |V(G)| = |V(H)|. It is easy to see that the class of planar graphs is pure. On the other hand, Harary has proved that the class of toroidal graphs is not pure. Therefore, we consider about the difference between pure classes and impure classes. A graph class \mathcal{G} is k-impure if $|E(G)| - |E(H)|| \leq k$ for all edge-maximal graphs $G, H \in \mathcal{G}$ with |V(G)| = |V(H)|. In 2016, McDiarmid and Wood have proved that the class of graphs embeddable in a surface of Euler genus g is O(g)-impure.

In our talk, we shall improve the above theorem for the projective planar case. (Friday 4:40)

Inplace multiplicity identities for integer compositions

<u>Augustine O. Munagi</u>* and James A. Sellers

* University of the Witwatersrand

In this talk, we give two new identities for compositions, or ordered partitions, of integers. These identities are based on a seamless analog of a recently studied identity in integer partitions. Using the structure inherent in integer compositions, we provide extensive generalizations of both identities. Bijective proofs are given and generating functions are provided for each of the types of compositions which arise. We close by discussing some arithmetic properties satisfied by the functions which count such compositions (Monday 10:00)

$Y\Delta$ -equivalence classes of projective planar maps and rhombus tilings of polygons

<u>Atsuhiro Nakamoto</u>* and Yuta Omizo * *Yokohama National University*

A map G on a surface F^2 is k-representative if every noncontractible closed curve on F^2 hits G at least k times. We say that G is k-minimal if G is k-representative, but every proper minor of G is not k-representative. Randby proved that for any k>0, any two k-minimal projective-planar maps are $Y\Delta$ -equivalent, i.e., transformed into each other by a repeated application of $Y\Delta$ exchanges.

In this talk, we first give another proof to this result, by using our result proving that every k-minimal projective-planar map is expressed as a rhombus tiling of a regular 2k-gon, by the notion of "radial graphs". Secondly, using the rhombus tilings of point-symmetric polygons, we find another $Y\Delta$ -equivalence class of projective-planar maps. (Thursday 4:15)

Stable embeddings of graphs on closed surfaces with respect to minimum length and their duals

Seiya Negami*

* Yokohama National University

Let G be a graph embedded on a closed surface F^2 which has a suitable metric. Then we can measure the length of each edge in G according to the metric. Denote the total summation of lengths of edges in G by ||G|| and call it the *total length* of G. The embedding of G is said to be *stable* (with respect to minimum length) if it satisfies the following two conditions:

- (i) The infimum taken over the total lengths of embedded graphs isotopic to G is attained by an embedding of G, say G_{\min} .
- (ii) If $||G_n||$ is convergent to $||G_{\min}||$, then a sequence of embeddings G_1, G_2, \ldots isotopic to G is convergent to an embedding of G.

For example, if G has a local planar part which shrinks to a point on F^2 , then it will not be stable since Condition (i) does not hold. We shall show a sufficient condition for a graph embedding to be stable and discuss the stableness of its dual embeddings. (Thursday 4:40)

An equivalence relation on shift sequences

 $\frac{\text{Kirsten Nelson}^*, \, \text{Daniel Panario}, \, \text{Brett Stevens}}{^*Carleton \, \, University}$

Interleaved sequences over a finite field in array form have columns that are either zero columns or phase shifts of a shorter sequence. Shift sequences, introduced by Games (1985), give a compact representation of an interleaved sequence, given the base sequence. We are interested in maximal-length interleaved sequences that are 'unique' in the sense that they create interleaved sequences that are not shift-equivalent to each other. It is known that the period of an interleaved sequence is dependent on the length of the shift sequence (Gong, 1995). We present the exact conditions for a given period to exist. Furthermore, we propose an equivalence relation that divides shift sequences into classes with unique representatives.

- R. A. Games. "Cross-correlation of m-sequences and GMW-sequences with the same primitive polynomial," *Discrete Applied Mathematics*, vol. 12, pp. 139-146, 1985. (Tuesday 10:00)
- G. Gong. "Theory and applications of q-ary interleaved sequences," *IEEE Transactions on Information Theory*, vol. 41, pp. 400-411, 1995.

Relationships between independence and matching coefficients of trees

Boon Leong Ng*

*National Institute of Education, Nanyang Technological University

We explore some ways of expressing the independence and matching coefficients of a tree in terms of the number of subgraphs, and derive some relations between the independence and matching coefficients. (Thursday 10:00)

Vf-safe delta-matroids

Joe Bonin, Carolyn Chun, <u>Steve Noble</u>*, Irene Pivotto and Gordon Royle $*Birkbeck,\ University\ of\ London$

Delta-matroids generalize ribbon graphs (or cellularly embedded graphs) in the same way that matroids generalize (abstract) graphs. In a ribbon graph, in addition to the standard way of forming a dual, there is a second notion of duality, corresponding to putting a half-twist in an edge. This operation corresponds to a natural operation on set systems, but when applied to an arbitrary delta-matroid may not yield a delta-matroid. A delta-matroid is vf-safe if every sequence of duality operations applied to it results in a delta-matroid. The class of vf-safe delta-matroids is closed under minors, but seems hard to characterize by excluded minors. We describe two results in this direction. (Thursday 5:05)

Triad colorings of triangulations on the torus

Yumiko Ohno*

* Yokohama National University

A triangulation on a closed surface is a graph embedded on the surface each of whose face is triangular. Let G be a triangulation on a closed surface and $n \ge 3$ be a natural number. A coloring $c: V(G) \to \mathbb{Z}_n$ is called an n-triad coloring if $\{c(u), c(v), c(w)\}$ belongs to $\{\{i, i+1, i+2\} \mid i \in \mathbb{Z}_n\}$ for any face uvw of G. Any n-triad coloring necessarily becomes ordinary an n-coloring. We would like to determine the set of numbers n such that G has n-triad colorings.

It is easy to show that if G has an n-triad coloring for $n \neq 4$, then G is an even triangulation, that is, each vertex of G has even degree. In particular, since the average degree of a triangulation on the torus is equal to 6, any even triangulation on the torus is either a 6-regular or has vertices of degree 4. It has been known that any even triangulation on the torus can be transformed into either a 6-regular one or one of 23 specified ones irreducible with respect to two reductions, called a 4-contraction and a twin-contraction, which eliminate one or two vertices of degree 4.

In fact, we can carry out these reductions, preserving n-triad colorings. We shall show that any even triangulation on the torus which can be transformed into one of the 23 irreducible even triangulations has chromatic number at least 4 and that the desired set of numbers is $\{4\}$ or $\{4,5\}$ if $\chi(G)=4$, and is the empty set otherwise, where $\chi(G)$ stands for the chromatic number of G. On the other hand, there exists an even triangulation G which can be transformed into 6-regular ones

and which has an arbitrarily large set of numbers n such that G has an n-triad coloring. (Thursday 5:30)

A characterization of testable hypergraph properties

Felix Joos, Jaehoon Kim, Daniela Kühn and <u>Deryk Osthus</u>*

* University of Birmingham

We provide a combinatorial characterization of all testable properties of k-graphs (i.e. k-uniform hypergraphs). Here, a k-graph property \mathcal{P} is testable if there is a randomized algorithm which makes a bounded number of edge queries and distinguishes with probability 2/3 between k-graphs that satisfy \mathcal{P} and those that are far from satisfying \mathcal{P} . For the 2-graph case, such a combinatorial characterization was obtained by Alon, Fischer, Newman and Shapira. Our results for the k-graph setting are in contrast to those of Austin and Tao, who showed that for the somewhat stronger concept of local repairability, the testability results for graphs do not extend to the 3-graph setting. Our proof makes use of the hypergraph regular approximation lemma due to Rödl and Schacht. (Tuesday 10:00)

Connectivity of cubical polytopes

Hoa Bui Thi, <u>Guillermo Pineda-Villavicencio</u>* and Julien Ugon ${}^*Federation\ University\ Australia$

A cubical polytope is a polytope with all its facets being combinatorially equivalent to cubes. The first part of the paper deals with the connectivity of the graphs of cubical polytopes. We first establish that, for any $d \ge 3$ and any $0 \le \alpha \le d-3$, the graph of a cubical d-polytope with minimum degree $d + \alpha$ is $(d + \alpha)$ -connected. Secondly, we show that, for any $d \ge 4$, any $0 \le \alpha \le d-3$ and any pair $(d, \alpha) \ne (4, 1)$, every separator of cardinality $d + \alpha$ in the graph of a cubical d-polytope consists of all the neighbours of some vertex and breaks the polytope into exactly two components.

The second part of the paper deals with the stronger concept of linkedness. A graph with at least 2k vertices is k-linked if, for every set of 2k distinct vertices organised in arbitrary k pairs of vertices, there are k disjoint paths joining the vertices in the pairs. Larman and Mani in 1970 proved that the graphs of simplicial d-polytopes, polytopes with all its facets being simplices, are $\lfloor (d+1)/2 \rfloor$ -linked; this is the maximum possible linkedness given the facts that a k-linked graph is at least (2k-1)-connected and that some of these graphs are d-connected but not (d+1)-connected. Here we establish that the graphs of cubical d-polytopes are also $\lfloor (d+1)/2 \rfloor$ -linked, for every $d \neq 3$; this is again the maximum possible linkedness. (Tuesday 11:50)

A Study of Imprimitive Half-Arc-Transitive Graphs

Nemanja Poznanović*

* University of Melbourne

A G-vertex-transitive graph Γ is said to be vertex-imprimitive if it admits a non-trivial G-invariant partition \mathcal{B} of its vertex set. This partition induces a quotient graph $\Gamma_{\mathcal{B}}$ which is also G-vertex-transitive.

Imprimitive G-arc-transitive graphs have been actively studied in recent decades. The goal of this research often being the reconstruction of a G-arc-transitive graph Γ from its imprimitive quotient $\Gamma_{\mathcal{B}}$ and the action of G on \mathcal{B} . Often this reconstruction is possible using a cross-sectional geometry which is naturally associated with an imprimitive G-arc-transitive graph.

In this talk, we will show how this 'geometric approach' can be extended to the study of imprimitive G-arc-transitive digraphs, and consequently, to the study of imprimitive G-half-arc-transitive graphs. Following the work done for arc-transitive graphs, we will show how certain G-half-arc-transitive graphs may be reconstructed from their imprimitive quotients. We will discuss the general methodology used here and highlight where it differs from the arc-transitive case. We consider several examples and case studies and also suggest some possible directions for future research. (Friday 12:05)

Composition lengths of finite groups

Stephen Glasby, Cheryl E Praeger*, Kyle Rosa and Gabriel Verret ${}^*University\ of\ Western\ Australia}$

Anne Penfold Street Memorial Session

The composition length c(G) of a finite group G is the number of its composition factors (counting multiplicities), and is sometimes viewed as a measure of its size or complexity. Various upper bounds on c(G) have been used to bound the size of a minimum generating set for G, or an invariable generating set for G, and there are various links with graph isomorphism. Often it is useful to have bounds in terms of parameters for the way the group is represented: for example, in terms of the degree of a permutation group, or the dimension and field size of a matrix group over a finite field.

My colleagues Stephen Glasby, Kyle Rosa and Gabriel Verret and I have found explicit, upper bounds for c(G), both for permutation groups and matrix groups G. The bounds are sharp and we have been able to describe all the examples which attain the bounds. Our bounds improve various results in the literature dating back to 1974. In particular, for primitive permutation groups G of degree n, where Pyber had obtained a ' $c \log n$ ' upper bound for c(G), we show that the optimal value for the constant $c \in 8/3$. (Friday 10:00)

Cubic edge-transitive bi-p-metacirculants

Yanli Qin* and Jinxin Zhou

* University of Melbourne & Beijing jiaotong University

A graph is said to be a bi-Cayley graph over a group H if it admits H as a group of automorphisms acting semiregularly on its vertices with two orbits. For a prime p, we call a bi-Cayley graph over a metacyclic p-group a bi-p-metacirculant. In this paper, the automorphism group of a connected cubic edge-transitive bi-p-metacirculant is characterized for an odd prime p, and the result reveals that a connected cubic edge-transitive bi-p-metacirculant exists only when p=3. Using this, a classification is given of connected cubic edge-transitive bi-p-metacirculants. As a result, we construct the first known infinite family of cubic semisymmetric graphs of order twice a 3-power. (Monday 3:50)

Combinatorics of tenth-order mock theta functions

M Rana* and JK Sareen

* Thapar University, Patiala, India

In this work, we provide the combinatorial interpretations of two tenth—order mock theta functions which appeared in some identities given in S. Ramanujans lost notebook. (Friday 4:40)

Some variants of the Erdős-Sós Conjecture

Gwenael Joret, Tash Morrisson, Jon Noel, <u>Bruce Reed</u>*, Alex Scott, Maya Stein, David Wood, Hehui Wu

* CNRS and McGill University

The Erdős-Sós Conjecture states that if a graph has average degree exceeding t-2 then it contains every tree with t vertices as a subgraph. This result was proved by Simonovitz and Szemeredi for large t. We discuss some variants of the conjecture including those obtained by replacing average degree with minimum degree and/or replacing subgraph with minor. (Saturday 11:00)

On the structure of thin cubic Lehman matrices

Dillon Mayhew, Irene Pivotto, <u>Gordon Royle</u>*

* University of Western Australia

Two $n \times n$ (0,1)-matrices A and B are called a Lehman pair if $AB^T = J + kI$, where J is the all-ones matrix and k is an integer. In this situation, it can be shown that both A and B have constant row and column sums, which we denote r and s respectively, and that k = rs - n. A matrix is called a Lehman matrix if it is part of a Lehman pair, and is called a thin Lehman matrix if it is part of a Lehman pair with k = 1. Research on Lehman matrices dates back more than five decades, largely due to the intimate connection between Lehman matrices and the still-unsolved problem of characterising minimally non-ideal matrices (a concept related to "packing and covering"). Thus there are a number of papers in the literature elucidating the structure of Lehman matrices, constructing families of Lehman matrices, and presenting computational studies of Lehman matrices with small n and small n.

In this talk I will focus on Lehman matrices with r = 3, which we dub *cubic Lehman matrices*, justifying our terminology by noting that a (0,1)-matrix with n rows and columns, and with row-sum and column-sum equal to 3 can be viewed as the bipartite adjacency matrix of a cubic bipartite graph on 2n vertices. The graphical viewpoint leads to more than just terminology, because interpreting the properties of a Lehman matrix in graphical terms allows us to isolate and identify unexpected structural properties of cubic Lehman matrices.

Our strongest structural results apply only to *thin* cubic Lehman matrices, but as only a finite number of non-thin examples are known, this is not a significant constraint. In particular, we give a unified construction that produces almost all known thin cubic Lehman matrices, isolates an intriguing omission in the literature, and provides a tantalising conjecture whose resolution would yield a more-or-less-complete characterization of thin cubic Lehman matrices.

This talk will be accessible to anyone with basic knowledge of graph theory and elementary linear algebra.

(Thursday 12:15)

The Rainbow Connection Number of Join of Some Graphs

A. Rosmanita and A. N. M. Salman*

*Institut Teknologi Bandung

Let G = (V, E) be a nontrivial connected graph and k be a positive integer, define a k-coloring c from E to $\{1, 2, ..., k\}$, where adjacent edges may be colored the same. A path P in G is called a rainbow path if no two edges of P are colored the same. A rainbow k-coloring is a-k-coloring such that every two distint vertices are connected by a rainbow path. Rainbow connection number of G denoted by rc(G) is the minimum k so that G has a rainbow k-coloring.

In this paper, we provide a lower bound and an upper bound for rainbow connection number of join of two graphs. We also determine the rainbow connection number of join of a complete graph with some graphs. (Tuesday 5:30)

Exponentially many nowhere-zero Z_3 -, Z_4 -, and Z_6 -flows

Zdeněk Dvořák, Bojan Mohar, and <u>Robert Šámal</u>*

* Charles University

We prove that, in several settings, a graph has exponentially many nowhere-zero flows. These results may be seen as a counting alternative to the well-known proofs of existence of \mathbb{Z}_3 -, \mathbb{Z}_4 -, and \mathbb{Z}_6 -flows. In the dual setting, proving exponential number of 3-colorings of planar triangle-free graphs is a related open question due to Thomassen. As a part of the proof we obtain a new splitting lemma for 6-edge-connected graphs, that may be of independent interest.

A sample of our results:

- If G is a 3-edge-connected graph with n vertices, then G has at least $2^{n/7}$ nowhere-zero $\mathbb{Z}_2 \times \mathbb{Z}_3$ -flows.
- If G is a 4-edge-connected graph with n vertices, then G has at least $2^{n/250}$ nowhere-zero \mathbb{Z}_2^2 -flows.
- Every 6-edge-connected graph G with n vertices has at least $2^{n/1540}$ nowhere-zero \mathbb{Z}_3 -flows.

(Friday 3:00)

A characterization of tree-tree quadrangulations on closed surfaces

Raiji Mukae and Terukazu Sano*

*National Institute of Technology, Kisarazu College

A graph G is said to be tree-tree if there is a vertex partition $V(G) = V_1 \cup V_2$ such that V_i induces a tree for i = 1, 2. This terminology is first introduced by Z.Skupień as "tree-tree triangulation" for studying the dual form of Barnette conjecture and the vertex arboricity for planar graphs. G.Schaar and Z.Skupień focus on constructing tree-tree triangulations on the plane. It is easy to see that there is no tree-tree triangulations on non-spherical closed surfaces.

In this talk, we will show that a characterization of tree-tree quadrangulations on non-spherical closed surfaces with special 4-regular multigraphs in the dual, and the specific 4-regular multigraphs for the projective plane and the torus. (Friday 4:15)

On local metric dimension of Mycielski graph

Suhadi Wido Saputro* and Nur Fahri Tadjuddin

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Indonesia

A set of vertices W locally resolves a connected graph G if every two adjacent vertices is uniquely determined by its distances coordinate to some vertices in W. The minimum cardinality of a local resolving set of G is called the local metric dimension of G, denoted by lmd(G). In this paper, we consider a graph product, namely Mycielski graph, denoted by $\mu(G)$. We investigate the relation between $lmd(\mu(G))$ and lmd(G) for any connected graphs G. We also characterize all connected graphs G such that $lmd(\mu(G)) = k$ for some values of positive integer k. (Friday 10:00)

Distance sets over finite spaces and finite Euclidean graphs

Shohei Satake*

*Kobe University

In combinatorics, there are some problems related to the Euclidean distance. One of such problems is the Erdős distance problem which asks the minimum size of the set of distances for given finite sets of points. While this problem has been researched, the analogous problem for finite spaces like F_q^n has also been investigated by many researchers. Here, as an analogue of the Euclidean distance, the quadratic form $d(x,y) = (x_1 - y_1)^2 + \cdots + (x_n - y_n)^2$ over F_q is usually adopted. Now we consider another (similar but different) problem which appears in algebraic combinatorics and other related areas. A s-distance set over the Euclidean space is a finite set of points that determine exactly s different Euclidean distances. The original problem, proposed by P. Erdős, is to find the maximum size of s-distance sets. When we define s-distance sets over F_q^n by using d(x,y), the same problem naturally appears.

In this talk, we give an upper bound of the maximum size of s-distance sets over finite fields. The proof is based on a graph-theoretic method and it uses the fact of the spectrum of finite Euclidean graphs defined by Medrano et.al. Moreover, we give some generalizations of s-distance sets over F_q^n or other finite spaces like finite upper half planes. (Monday 11:50)

Gaussian designs, quasi-Hermite polynomials, Hausdorff-type Diophantine equations

Masanori Sawa*

*Kobe University

My talk will kick off with an introduction of a close connection between Gaussian designs in algebraic combinatorics and the zeros of quasi-Hermite polynomials in special function. We also consider a certain system of Diophantine equations, originally designed by Hausdorff (1909) as a simplification of Hilbert's solution of Waring's problem, and then create the relationship to Gaussian designs and quasi-Hermite polynomials. We reduce these equations to the existence problem of rational points on a hyperelliptic curve associated with the discriminant of a quasi-Hermite polynomial, and thereby show a nonexistence theorem for solutions of Hausdorff-type equations. For this purpose, we enjoy an elegant explicit formula for the discriminant of a quasi-Hermite polynomial, which is a natural generalization of Hilbert's formula on the discriminant of a Hermite polynomial (1888). This is a joint work with Yukihiro Uchida (Tokyo Metropolitan University). (Thursday 5:30)

Permutations and recurrent configurations of the Abelian sandpile model on Ferrers graphs

Mark Dukes, Thomas Selig*, Jason P. Smith and Einar Steingrímsson *University of Strathclyde, Glasgow, UK

The Abelian sandpile model (ASM) is a dynamic process on a graph G. More precisely, it is a Markov chain on the set of configurations on G. Ferrers graphs are a class of bipartite graphs which are in one-to-one correspondence with Ferrers diagrams. We study the ASM on Ferrers graphs. We establish a bijection between the set of minimal recurrent configurations and a set of permutations whose descent bottoms set is given by the structure of the Ferrers graph. We describe a certain decoration of these permutations and show that there is a one-to-one correspondence between these decorated permutations and the set of all recurrent configurations on the Ferrers graph. (Friday 3.50)

Matroids and Codes with the Rank Metric

Keisuke Shiromoto*

 $*Kumamoto\ University,\ Japan$

A (Delsarte) rank-metric code of size $n \times m$ over a finite field \mathbb{F}_q is an \mathbb{F}_q -linear subspace of the $n \times m$ matrix space $\operatorname{Mat}(n \times m, \mathbb{F}_q)$ over \mathbb{F}_q . We denote by \mathcal{S}_n the set of all subspaces of the n-dimensional vector space \mathbb{F}_q^n . A (q, r)-polymatroid is a q-analogue of k-polymatroids, and is defined by an ordered pair $\mathcal{M}_q = (\mathbb{F}_q^n, \rho)$ consisting of \mathbb{F}_q^n and a function $\rho : \mathcal{S}_n \to \mathbb{Z}^+ \cup \{0\}$ having the following properties:

- (R1) If $A \in \mathcal{S}_n$, then $0 \leqslant \rho(A) \leqslant r \dim A$.
- (R2) If $A, B \in \mathcal{S}_n$ and $A \subseteq B$, then $\rho(A) \leqslant \rho(B)$.
- (R3) If $A, B \in \mathcal{S}_n$, then $\rho(A+B) + \rho(A \cap B) \leq \rho(A) + \rho(B)$.

In this talk, we show a relationship between a rank-metric code over a finite field and a (q, r)polymatroid. We give a Greene type identity for the rank generating function of these polymatroids
and the rank weight enumerator of these codes. Finally, we prove a MacWilliams type identity on
codes with rank metric. (Thursday 4:40)

Restricted Size Ramsey Number for $2K_2$ versus Dense Connected Graphs

Denny Riama Silaban*, Edy Tri Baskoro, Saladin Uttunggadewa

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Let G and H be simple graphs. The size Ramsey number $\hat{r}(G, H)$ is the smallest number \hat{r} such that there exists a graph F with size \hat{r} satisfying the property that any 2-coloring of the edges of F contains a monochromatic G or H. If the order of F in the size Ramsey number equals to the Ramsey number, r(G, H), then it is called the restricted size Ramsey number $r^*(G, H)$. In 1981, Erdös and Faudree gave the size Ramsey number involving matching. However, it is not for the restricted one. Recently, we characterized all graph with no isolates H for which $r^*(2K_2, H)$ attaining known upper and lower bounds. We also gave the restriction version for some results involving $2K_2$ from Erdös and Faudree. In this work, we continue the investigation on $r^*(2K_2, H)$. We give $r^*(2K_2, H)$ for H are some connected dense graphs. (Thursday 11:50)

The Multiset Dimension of Graphs

Rinovia Simanjuntak*, Tomáš Vetrík, and Presli Bintang Mulia

*Institut Teknologi Bandung

We introduce a variation of the metric dimension, called the multiset dimension.

The representation multiset of a vertex v with respect to W (which is a subset of the vertex set of a graph G), $r_m(v|W)$, is defined as a multiset of distances between v and the vertices in W together with their multiplicities. If $r_m(u|W) \neq r_m(v|W)$ for every pair of distinct vertices u and v, then W is called a resolving set of G. If G has a resolving set, then the cardinality of a smallest resolving set is called the multiset dimension of G, denoted by md(G). If G does not contain a resolving set, we write $md(G) = \infty$.

We present basic results on the multiset dimension. We give some sufficient conditions for a graph to have an infinite multiset dimension. We also study the multiset dimension of trees and Cartesian product graphs.

(Tuesday 3:00)

Almost 2-transitive finite elation Laguerre planes

Günter Steinke*

* University of Canterbury

Finite Laguerre planes of order n are precisely the transversal designs $TD_1(3, n + 1, n)$. All known finite Laguerre planes are ovoidal, that is, they are obtained as the geometry of non-trivial plane sections of a cone over an oval in 3-dimensional projective space. It is a longstanding open problem whether or not these are the only models of finite Laguerre planes.

Elation Laguerre planes generalize the notion of ovoidal Laguerre planes and are characterized by the existence of a group of automorphisms that acts trivially on the set of generators and regulary on the set of circles (the blocks of the transversal design). They are equivalent to translation generalized quadrangles and are obvious candidates to look for non-ovoidal finite Laguerre planes. It is natural to impose additional homogeneity conditions at first. It is known that if the automorphism group of a finite elation Laguerre plane is 2-transitive on the set of generators or is non-solvable and acts primitively, then the plane is ovoidal over a conic.

In this talk we consider finite elation Laguerre planes whose automorphism groups fix a generator and are 2-transitive on the set of remaining generators. One obtains that derived projective planes at points on the distinguished generator have Lenz type at least V. Moreover, they must be Desarguesian in case of odd order — the Laguerre plane then is ovoidal over a conic and the associated generalized quadrangle is the classical orthogonal generalized quadrangle Q(4, q). In even order and a Desarguesian derivation one obtains ovoidal Laguerre planes over translation ovals. This occurs when the order is 2^m where m is either a prime number or the square of a prime number. The smallest unresolved case is order 64. (Tuesday 4:15)

Balanced equi-n-squares: a generalization of Latin squares

Saieed Akbari, <u>Rebecca J. Stones</u>*, Zhuanhao Wu *Nankai University

Equi-n-squares are a generalization of Latin squares: they are $n \times n$ matrices that contain n distinct symbols in which each symbol occurs exactly n times. We discuss a special case of equi-n-squares: those for which, for some divisor d of n, any symbol that occurs in a row or column, occurs exactly d times in that row or column. We call these d-balanced equi-n-squares, and Latin squares are when d = 1. In graph theory, a d-balanced equi-n-square is equivalent to a decomposition of $K_{n,n}$ into d-regular spanning subgraphs of $K_{n/d,n/d}$.

We identify how d-balanced equi-n-squares L can be constructed from a partition of a Latin square of order n into $d \times (n/d)$ subrectangles. We also study when L is diagonally cyclic, which corresponds to cyclic decompositions of $K_{n,n}$.

We identify necessary conditions for the existence of (a) d-balanced equi-n-squares, (b) diagonally cyclic d-balanced equi-n-squares, and (c) Latin squares of order n which partition into $d \times (n/d)$ subrectangles. We discuss our progress on identifying when the necessary conditions are sufficient. (Tuesday 4:40)

On Inclusive and non Inclusive Distance Vertex Irregular Labelings

Kiki Ariyanti Sugeng*

* Universitas Indonesia

Let G be a simple graph. A vertex labeling $f:V(G)\to\{1,2,\ldots,k\}$ of G is called k-labeling. A non inclusive distance vertex irregular labeling of a graph is a mapping $g,g:V(G)\to\{1,2,\ldots,k\}$ such that the set of vertex weights consists of distinct numbers, where the weight of a vertex $v\in V(G)$ under the labeling g is defined as the sum of all vertex labels of vertices in the open neighborhood of the vertex v While an inclusive distance vertex irregular distance k-labeling of G is a vertex k-labeling and for every two different vertices u and v there is $wt_f(u)\neq wt_f(v)$, where $wt_f(v)$ is the sum of all vertex labels of vertices in the closed neighborhood of the vertex v. The (non)inclusive distance vertex irregularity strength of G is the minimum k for which the graph G has a vertex irregularity strength for several families of graphs. (Friday 12:05)

The Bisection Width of the Levi Graph of PG(2,q)

Muhammad Adib Surani*

* University of Melbourne

The bisection width of a graph is the minimum number of edges whose deletion splits the graph into two equal sized parts (up to one vertex). The problem of determining this number is NP-complete in general, but we will establish some bounds on this value for the incidence graph of PG(2, q).

At the heart of this talk is an investigation of the quantity

$$M(q) := \max_{S \subseteq \mathbb{F}_q} \sum_{t \in \mathbb{F}_q} \left| \sum_{s \in S} \chi(s+t) \right|,$$

where q is an odd prime power and χ is the quadratic character of \mathbb{F}_q . We will construct a lower bound on M(q) using a generalisation of the tangent numbers due to Shanks (1967). To that effect, we will explore some beautiful number theoretical results involving Dirichlet L-functions, Euler products, Fourier series, and Gauss sums, before finally using M(q) to bound the bisection width.

If time permits, we might also look at the point-hyperplane incidence graph of PG(n,q) for larger n. (Tuesday 5:05)

The Most Frequent Connected Induced Subgraph problem

<u>Srinibas Swain</u>*, Graham Farr, Kerri Morgan, Paul Bonnington

*Monash University

The frequency of an unlabelled graph H in a graph G is the number of induced subgraphs of G that are isomorphic to H. The graph H is a most frequent connected induced subgraph (MFCIS) of G if the frequency of H in G is maximum among all unlabelled graphs occurring as induced subgraphs of G. We introduce the MFCIS problem and discuss some results on it including its complexity. We found by computation that the MFCIS of all non-clique graphs of up to 12 vertices is always one of the following graphs: K_2, K_3, P_2, P_3 , where P_k denotes the path graph with k edges. All these graphs are of order at most 4, however we give an infinite class of graphs whose MFCIS contains 75% of the vertices of the graph. We also determine MFCIS of some special classes of graphs. (Friday 3:00)

Patterns in Arc System

 $\frac{\text{Murray Tannock}^* \text{ and Michael Albert}}{^* \textit{University of Otago}}$

An Arc System of size n is a collection of n arcs joining 2n points on a horizontal line. Borrowing ideas and notation form the field of permutation patterns we we recall and extend definitions of patterns in arc systems given by Bloom and Elizalde, first defining a "classical" arc system pattern and then defining a more general arc system pattern analogous to mesh patterns in a permutation. Computational experiments then allow us to classify classes of arc systems defined by avoidance of generalised arc patterns with a single arc by both coincidence and Wilf-equivalence; and provide us with evidence to draw some general conclusions on this type of pattern, such as when a given arc pattern is unsatisfiable. (Friday 4:15)

A generic approach to switching reconstruction

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As an example of a switching operation, let \mathcal{G} be the set of 2-vertex-coloured simple graphs on a vertex set Z, with two elements of \mathcal{G} being equivalent if they are isomorphic as coloured graphs. For $G \in \mathcal{G}$, define switching at $W \subseteq Z$ to mean swapping the colour of all vertices in W and denote the resulting graph by G_W . The deck of G is the formal sum $\sum_{z \in Z} G_{\{z\}}$. We say G is reconstructible with respect to this switching operation if it is uniquely determined up to equivalence by its deck.

In this talk, we define a generalised switching operation that encompasses the one described above as well as familiar examples such as Stanley's vertex-switching and Bondy and Mercier's digraph switching. We then discuss some generic results and illustrate how they can be applied to specific switching reconstruction problems. In particular, we show that if $G \in \mathcal{G}$ is not reconstructible, then there must be at least $2^{|Z|/2-1}$ sets $W \subset Z$ for which $G_W \sim G$. This enables us to derive a generic sufficient condition for reconstructibility and prove, for example, that if in the digraph switching problem the underlying graph of G is a 3-connected planar graph of order at least 13, then G is reconstructible.

(Thursday 3:00)

Hadamard difference sets of order 256

John J. Cannon and <u>Don Taylor</u>*

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A k-subset D of a group G of order v is a (v, k, λ) -difference set if $DD^- = (k - \lambda)1_G + \lambda G$ in the integral group ring $\mathbb{Z}[G]$, where subsets of G are identified with their sum in $\mathbb{Z}[G]$ and D^- is the sum of the inverses of the elements of D.

The sets Dg are the blocks of a (v, k, λ) -design with incidence matrix N. If J is the all-one matrix, J - 2N is a Hadamard matrix if and only if $(v, k, \lambda) = (4m^2, 2m^2 - m, m^2 - m)$ for some m. When G is a 2-group any difference set in G must have these parameters with $m = 2^d$ for some d.

There are 56 092 groups of order 256 and in 2010 J. F. Dillon proposed using MAGMA to determine which of these groups contain a difference set. At the time of writing there are 43 known exceptions and 28 groups yet to be decided. This talk reports on the various methods used to construct difference sets in these 2-groups and the structure of the exceptions. (Wednesday 10:25)

Two New Statistics on Square Walks.

Lenny Tevlin*

*American University of Malta

This talk considers a q-generalization of the classical result on the number of square walks between two lattice points in the plane, a half-plane, and a quarter-plane D. DeTemple and J. Robertson (1984); R. K. Guy, C. Krattenthaler, and B.E. Sagan (1992). These are given by linear combinations of products of binomial coefficients.

To that end I introduce two new statistics on square walks, generalizations of inversion and major index statistics. Analogously to the classical result, the product of q-binomial coefficients is a generation function of these (proved to be equally distributed) statistics on square walks, with similar results about a half-plane and a quarter-plane. Additionally, it is conjectured that these statistics are equally distributed over walks with a fixed number of descents. (Thursday 3:00)

Distance Antimagic Labeling of Graph Products

Aholiab Tegar Tritama* and Rinovia Simanjuntak

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A graph G with order n is called distance antimagic if there exists a bijection from the set of vertices to the set of integers $1, 2, \ldots, n$ such that all vertex sums are pairwise distinct, where a vertex sum is the sum of the maps of all neighbours of that vertex. According to Kamatchi-Arumugam's conjecture, every graph without two vertices having the same neighborhood is distance antimagic.

In this paper, we present results on distance antimagic labeling for products of graphs, where the products are cartesian, corona, lexicographic and strong product. Our main tool is by defining a bijection for the product graph based on the bijection for the base graphs. (Friday 11:40)

On distance matching extendability of fullerene graphs

M. Furuya, M. Takatou and S. Tsuchiya*

*Senshu University

A fullerene graph is a 3-connected cubic plane graph whose all faces are bounded by 5- or 6-cycles. It has been known that fullerene graphs can be regarded as chemical graphs of fullerenes, where carbon atoms are represented by vertices of the graph, whereas the edges represent bonds between adjacent atoms. Furthermore, a perfect matching of a fullerene graph is corresponding to the Kekulé structure of a fullerene. By such a reason, some properties of perfect matchings in fullerene graphs have been widely studied. Recently, we proved that a matching M of a fullerene graph can be extended to a perfect matching if the following hold: (i) Three faces around each vertex in $\{x,y:xy\in M\}$ are bounded by 6-cycles and (ii) the edges in M lie at distance at least 13 pairwise. This result is related to the distance matching extendability of graphs. In this talk, we introduce the above result and related topics.

(Thursday 3:50)

Group Divisible Designs with $\lambda_1=3$ and Larger Second Index

Chariya Uiyyasathian* and Natapan Kitisin

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A group divisible design $\mathsf{GDD}(m,n;\lambda_1,\lambda_2)$ is an ordered pair (V,\mathcal{B}) where V is an (m+n)-set of symbols while \mathcal{B} is a collection of 3-subsets (called blocks) of V satisfying the following properties: the (m+n)-set is divided into 2 groups of size m and of size n: each pair of symbols from the same group occurs in exactly λ_1 blocks in \mathcal{B} : and each pair of symbols from different groups occurs in exactly λ_2 blocks in \mathcal{B} . λ_1 and λ_2 are referred to as the first index and second index, respectively. When $\lambda_1 \geq \lambda_2$, the existence problem of $\mathsf{GDD}(m,n;\lambda_1,\lambda_2)$ is completely solved for $m,n\neq 2$ in 2009-2012. The case $\lambda_1 < \lambda_2$ is considered more difficult, however recently some progress has been made when $\lambda_1 = 1$ and 2. This paper focuses on the existence problem of GDD s when $\lambda_1 = 3 < \lambda_2$. We obtain the necessary conditions and prove that these conditions are sufficient for most of the cases. (Thursday 5:05)

Distinguishing number of vertex-transitive graphs of valency 4

Gabriel Verret*

* University of Auckland

The distinguishing number of a graph G is the smallest number of colours needed to colour the vertices of G such that the only colour-preserving automorphism of G is the identity.

It is an easy exercise to show that cycles of length at least 6 have distinguishing number 2. Similarly, apart from finitely many exceptions, connected vertex-transitive graphs of valency 3 also have distinguishing number 2.

Recently, Florian Lehner and I determined the distinguishing number of all vertex-transitive graphs of valency 4. In this case, there are infinitely many examples with distinguishing number 3. I will discuss this recent work, as well as some intriguing related conjectures. (Thursday 3:00)

The 1-good-neighbor connectivity and diagnosability of Cayley graphs generated by complete graphs

Mujiangshan Wang*, Yuqing Lin, Shiying Wang

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Diagnosability is a significant metric to measure the reliability of multiprocessor systems. In 2012, a new measure for fault tolerance of the system was proposed by Peng et al. This measure is called the g-good-neighbor diagnosability that restrains every fault-free node to contain at least g fault-free neighbors. The Cayley graph CK_n generated by the complete graph K_n has many good properties as other Cayley graphs. In this paper, we show that the connectivity of CK_n is $\frac{n(n-1)}{2}$, the 1-good-neighbor connectivity of CK_n is $n^2 - n - 2$ and the 1-good-neighbor diagnosability of CK_n under the PMC model is $n^2 - n - 1$ for $n \ge 4$ and under the MM* model is $n^2 - n - 1$ for $n \ge 5$. (Monday 3:00)

Congruences for Permanents

Saieed Akbari, Darcy Best, <u>Ian Wanless</u>*

*Monash University

The permanent is matrix function (related to the determinant) that is useful in many counting problems. We consider permanents of integer matrices in classes of combinatorial interest, and ask whether there are any non-trivial congruences satisfied by those permanents. A sample result, phrased in graph theoretic terms, is this:

Theorem. Let G be a k-regular bipartite graph with n vertices in each partite set. If n is odd and $k \equiv 0 \mod 4$ then the number of perfect matchings in G is a multiple of 4.

We have a variety of theorems and conjectures in a similar vein. (Saturday 11:50)

The Slow-Coloring Game (Online Sum-Paintability)

Gregory Gutowski, Thomas Krawczyk, Thomas Mahoney, Gregory J. Puleo, <u>Douglas B. West</u>*, Michal Zajac, and Xuding Zhu

*Zhejiang Normal University and University of Illinois

The "slow-coloring game" is played by Lister and Painter on a graph G. On each round, Lister marks a nonempty subset M of the remaining vertices, scoring |M| points. Painter colors a subset of M that is independent in G. The game ends when all vertices are colored. Painter seeks to minimize the total score; Lister seeks to maximize it. The score under optimal play is the "slow-color cost" of G, written s(G). We describe various results on this parameter.

Trivial lower and upper bounds on s(G) are the chromatic sum and the sum-paintability, which are sharp. We give sharp upper and lower bounds on s(G) using the independence number. We have a linear-time algorithm to compute s(G) exactly when G is a tree; among n-vertex trees, it is minimized by the star and maximized by the path (where it equals 3n/2), We give good bounds on $s(K_{r,s})$ and conjecture $s(K_{r,r}) \sim 4r$. (Joint work with Thomas Mahoney and Gregory Puleo.)

For a d-degenerate graph G with n vertices, $s(G) \leq (1+3d/4)n$. The upper bound improves to $s(G) \leq 7n/3$ for outerplanar graphs and $s(G) \leq 3.9857n$ for planar graphs, with the conjectured optimal bounds being 2n and 5n/2, respectively. (Joint work with Grzegorz Gutowski, Tomasz Krawczyk, Michal Zajac, and Xuding Zhu.) (Monday 12:15)

Path-Dependent Colour Schemes

Tim E. Wilson*

*Monash University

A colour scheme is a set of vertex-coloured graphs. Many colour schemes exist in the literature in the form of a rule which determines whether a coloured graph is in the colour scheme. For example, the colour scheme corresponding to proper colouring is the set of finite vertex-coloured graphs with no monochromatic edges.

A colour scheme is *path dependent* if inclusion in the scheme depends only on avoiding some set of bad subpaths. Examples of path-dependent colour schemes include star colouring, non-repetitive colouring, anagram-free colouring and parity colouring. I generalise properties shared by many examples of path-dependent colour schemes and, for these properties, give sufficient conditions for a path-dependent colour scheme to be bounded on subdivisions of graphs. (Tuesday 4:15)

Bad News for Chordal Partitions

Alex Scott Paul Seymour <u>David R. Wood*</u>

*Monash University

Reed and Seymour [1998] asked whether every graph has a partition into induced connected nonempty bipartite subgraphs such that the quotient graph is chordal. If true, this would have significant ramifications for Hadwiger's famous graph colouring conjecture. We prove that the answer is 'no'. In fact, we show that the answer is still 'no' for several relaxations of the question. (Tuesday 4:40)

Vertex Partition with average degree constraint

Yang Wang, <u>Hehui Wu</u>*

*Shanghai Center for Mathematical Sciences, Fudan University

A well known result of Stibietz stated that given a graph G and two functions $s, t : V(G) \to N$ such that $d_G(v) \ge s(v) + t(v) + 1$, then there is a nontrivial vertex partition (A, B), such that $d_A(v) \ge s(v)$ for all $v \in A$ and $d_B(v) \ge t(v)$ for all $v \in B$. In this talk, we will report some results of two analogous open problems.

In a paper by Csóke, Lo, Norin, Yepremyan and the second author, it is conjectured that if G is a graph with $e(G) \ge (s+t+1)v(G)$ for non-negative real number s and t, then there exist a nontrivial vertex partition (A, B), such that $e(G[A]) \ge s|A|$ and $e(G[B]) \ge t|B|$. With Yan Wang, we proved that there exists two disjoint vertex set A and B that satisfied the required degree constraint.

Cyclotomic difference sets

Binzhou Xia*

* University of Melbourne

The study of cyclotomic difference sets dates back to Paley in 1930s ' and has become well-known for its connection to the long-standing conjecture that every finite flag-transitive projective plane is Desarguesian. In this talk, I will propose a new approach to this classic problem, which gives insights from character sums, p-adic gamma functions and systems of polynomial equations. (Wednesday 10:00)

On Constructing Normal and Non-Normal Cayley Graphs

Yian Xu*

*The University of Western Australia

Bamberg and Giudici (2011) showed that the point graphs of certain generalised quadrangles of order (q-1,q+1), where $q=p^k$ is a prime power with $p \ge 5$, are both normal and non-normal Cayley graphs for two isomorphic groups. We call these graphs BG-graphs. In this talk, we show that the Cayley graphs obtained from a finite number of BG-graphs by Cartesian product, direct product, and strong product also possess the property of being normal and non-normal Cayley graphs for two isomorphic groups. (Monday 4:40)

Polytopes with few edges are rare

Guillermo Pineda, Julien Ugon and <u>David Yost</u>*

*Federation University

It is known that a d-dimensional polytope with $v \leq 2d$ vertices has at least

$$\phi(v,d) = 2dv - d^2 - d - \frac{1}{2}v^2 + \frac{1}{2}v$$

edges, and that there is only one such polytope (up to combinatorial equivalence) with exactly $\phi(v,d)$ edges. We show that polytopes with $\phi(v,d)+1$ or $\phi(v,d)+2$ edges are quite rare. In particular, there are no polytopes with v vertices and $\phi(v,d)+1$ edges if $v\geqslant d+4$, and no polytopes with v vertices and $\phi(v,d)+2$ edges if $v\geqslant d+5$. For v=d+3, we completely characterise d-dimensional polytopes with v vertices and $\phi(v,d)+1$ or $\phi(v,d)+2$ edges. We offer partial results for the cases v=2d+1 and v=2d+2. (Tuesday 12:15)

Characterisations of Extended Tutte Invariants for Alternating Dimaps

Kai Siong Yow*, Graham Farr and Kerri Morgan
*Monash University

An alternating dimap is an orientably embedded Eulerian directed graph where the edges incident with each vertex are directed inwards and outwards alternately (Tutte, 1948). Three reduction operations on alternating dimaps were investigated by Farr (2013). A minor of an alternating dimap can be obtained by any sequence of these reductions. Unlike classical minor operations, these reduction operations do not commute in general.

A Tutte invariant for alternating dimaps is a function F defined on every alternating dimap and taking values in a field $\mathbb F$ such that F is invariant under isomorphism and obeys a certain linear recurrence relation involving reduction operations reminiscent of the relation satisfied by the Tutte polynomial. It is well known that if a graph G is planar, then $T(G; x, y) = T(G^*; y, x)$. We prove an analogous relation for an extended Tutte invariant for alternating dimaps. We then characterise the extended Tutte invariant under several conditions. As a result of the non-commutativity of the reduction operations, these Tutte invariants are not always well defined. We investigate the properties of alternating dimaps that are required in order to obtain a well defined Tutte invariant. We also give some excluded minor characterisations for alternating dimaps when their Tutte invariants are well defined. (Monday 4:40)

Properties of chromatics polynomials of hypergraphs not held for chromatic polynomials of graphs

Ruixue Zhang* and Fengming Dong

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In this talk, we shall present some properties on chromatic polynomials of hypergraphs which do not hold for chromatic polynomials of graphs. We first show that chromatic polynomials of hypergraphs have all integers as their zeros and contain dense real zeros in the set of real numbers. We then prove that for any multigraph G = (V, E), the number of totally cyclic orientations of G is equal to the value of $|P(\mathcal{H}_G, -1)|$, where $P(\mathcal{H}_G, \lambda)$ is the chromatic polynomial of a hypergraph \mathcal{H}_G which is constructed from G. Finally we show that the multiplicity of root "0" of $P(\mathcal{H}, \lambda)$ may be at least 2 for some connected hypergraphs \mathcal{H} , and the multiplicity of root "1" of $P(\mathcal{H}, \lambda)$ may be 1 for some connected and separable hypergraphs \mathcal{H} and may be 2 for some connected and non-separable hypergraphs \mathcal{H} . (Monday 4:15)

Point-transitivity of 2-(196, 6, 1) designs

Xiaohong Zhang* and Shenglin Zhou

*South China University of Technology

We investigate the automorphism group G of a putative point-transitive 2-(196, 6, 1) design and show that this group is a $\{2, 3, 5, 7\}$ -group which contains no subgroup of order 35. Furthermore, it is shown that the minimal normal subgroup of G can only be elementary abelian 2-groups, elementary abelian 7-groups, or simple groups PSL(2,7), PSL(2,8), PSU(3,3). (Thursday 3:50)

Perfect Codes in Cyclotomic Graphs

Sanming Zhou*

* The University of Melbourne

Let G = (V, E) be a graph and t a positive integer. A perfect t-code in G is a subset C of V such that the t-neighbourhoods of the vertices in C form a partition of V, where the t-neighbourhood of a vertex is the set of vertices within distance t from it. Perfect t-codes in Hamming graph H(n,q) are precisely q-ary perfect t-codes of length n in the classical setting. A perfect 1-code in a graph is also called an efficient dominating set or independent perfect dominating set of the graph.

I will talk about perfect 1-codes in two families of cyclotomic graphs, which are Cayley graphs on the additive group of $\mathbb{Z}[\zeta_m]/A$, with connection sets $\{\pm(\zeta_m^i+A):0\leqslant i\leqslant m-1\}$ and $\{\pm(\zeta_m^i+A):0\leqslant i\leqslant \phi(m)-1\}$, respectively, where ζ_m $(m\geqslant 2)$ is an mth primitive root of unity, A a nonzero ideal of $\mathbb{Z}[\zeta_m]$, and ϕ Euler's totient function. (Friday 3:50)

